

Machine Dynamics

Chapter M : Mass Properties



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Mass property of rigid body

- Determination of mass properties for a rigid body.

1) The center of mass :

Definition →

The point through which the
on the body acts of the position
of the body.

∴ If the total mass is M , the coordinates of the mass center are:

$$Mx_g = \int_M x dm, \quad My_g = \int_M y dm,$$



Mass property of rigid body (cont.)

a) Analytical methods :

For simple geometry or irregular shapes which are composed of common regular shapes.

- ① Most machine elements have two x and y axes, the center of mass lies at the intersection of these axes.

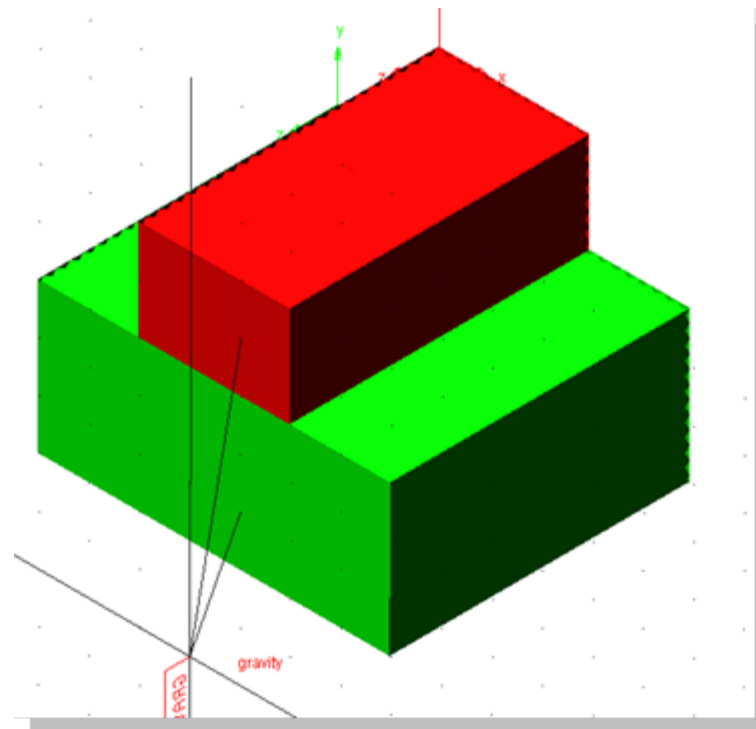
Example : flywheels, pulleys, gear, etc.

- ② Calculate complex geometry by dividing it into several small regular elements.
→ by hand calculations :



Mass property of rigid body (cont.)

→ by computer :
create models of machine parts by
operations & let the computer do
the calculations.





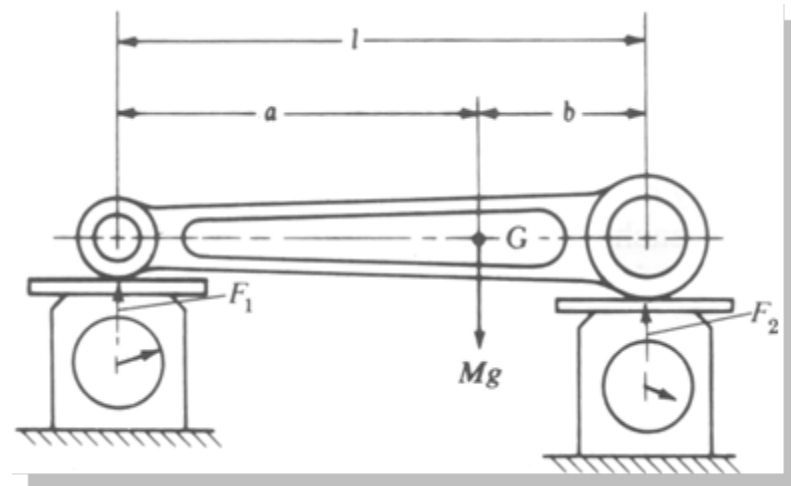
Mass property of rigid body (cont.)

b) Experimental methods :

- ① Suspend the body and let it free to rotate, draw a vertical line through the point of suspension. Choose another suspension point and draw another vertical line again. The point of these two lines locates the C.G.
- ② For machine parts with one

Example :

$$Mg \cdot a = F_2 \cdot l$$





Mass property of rigid body (cont.)

2) The moment of inertia :

Definition →

r : the distance from the axis of rotation to dm .

→ The parallel theorem:

: the moment of inertia whose axis is
to the axis of I_g .

: the moment of inertia whose axis passes
through the



Mass property of rigid body (cont.)

M : the total mass of the body.

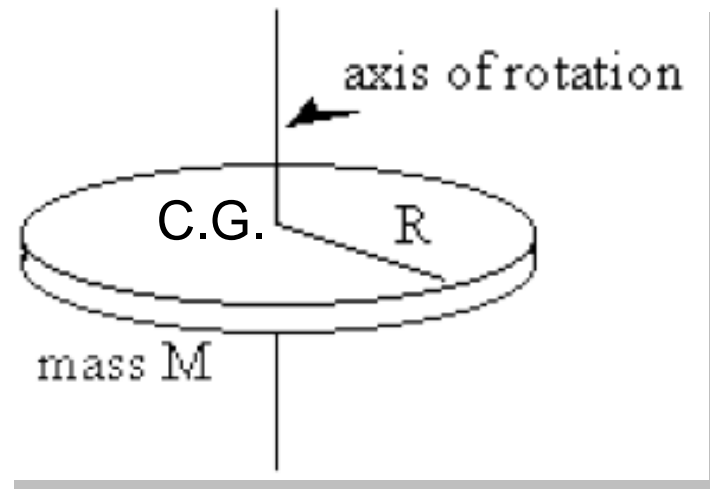
d : the distance between these two axes.

✘ Two important conditions:

① I_g is referred to the axis **through**

② the axes of I_g & I_o must be

✘





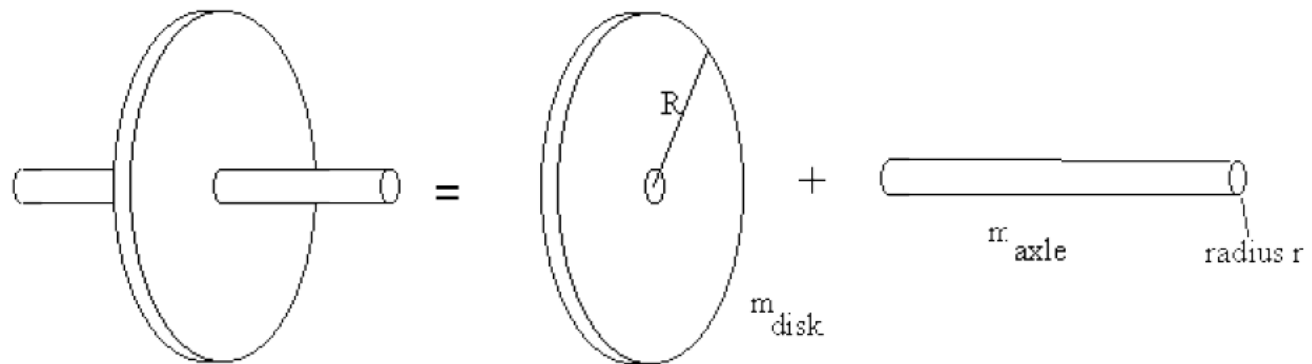
Mass property of rigid body (cont.)

a) Analytical methods :

the body into small segments whose I (through their own .) are known and apply **theorem** to calculate & sum up individual I to a specific axis of rotation.

① by hand calculations

② by computer (models)





Mass property of rigid body (cont.)

b) Experiment methods :

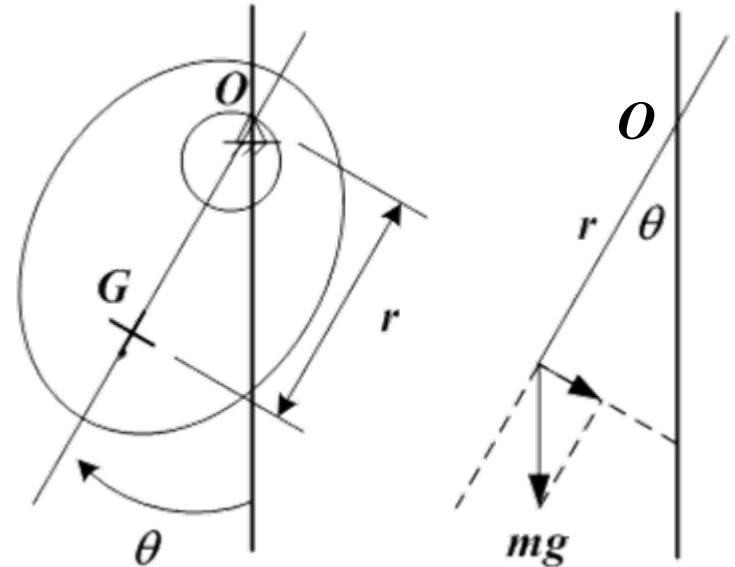
① The hanging method

The body is supported on a vertical line, at any point O other than the C.G.

Let the body swing with a small angle θ about O .

Observe the time required for a number of oscillations. “ I ” can then be calculated.

$$\sum M_0 = 0 \Rightarrow T_0 = I_0 \alpha$$





Mass property of rigid body (cont.)

$$-Mg \cdot r \sin \theta = I_0 \frac{d^2 \theta}{dt^2}$$

when θ is small \Rightarrow

$$\text{Assume } \theta = A \sin \sqrt{\frac{Mgr}{I_0}} t + B \cos \sqrt{\frac{Mgr}{I_0}} t$$

Initial conditions :

$$\text{at } t = 0, \quad \theta = \theta_{\max}, \quad \omega = \dot{\theta} = 0$$

$$\therefore A = 0, \quad B = \theta_{\max}$$



Mass property of rigid body (cont.)

The period T is :

$$\therefore I_0 =$$

→ The moment of inertia about the C.G. is:

$$I_g =$$

※ The accuracy is dependent upon **the accuracy of** . To insure the accuracy, it is desired to make or so that “ I ” is not seriously affected by the



Mass property of rigid body (cont.)

→ suspended at the other end is better in view of accuracy (∵).

Example :

遠端 60 sec內, count 58次或59次

→

近端 60 sec內, count 66次或65次

→

→ **This result is more**

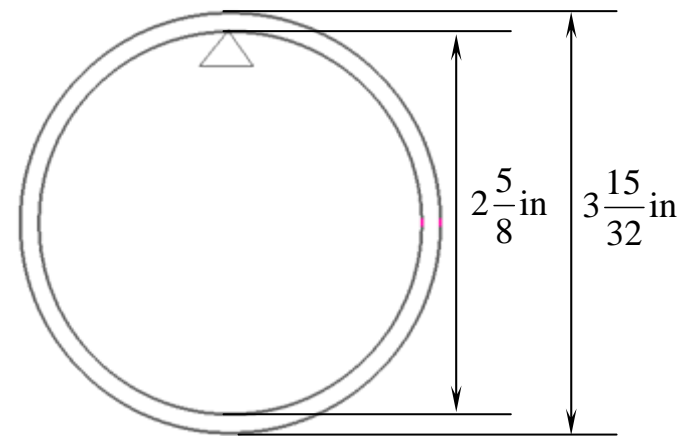
→ if “*O*” is too close to the C.G., it will be difficult to get an accurate time of oscillations due to
The system can oscillate only a few cycles.



Mass property of rigid body (cont.)

Example :

Determine the I for the ring whose weight is 1.203 lb & makes 107 oscillations in 1 min when supported as shown.



✓ by formula

$$I = \frac{1}{2} \cdot \frac{1.203}{g} \left[\left(\frac{2.625}{2} \right)^2 + \left(\frac{3.469}{2} \right)^2 \right]$$

=



Mass property of rigid body (cont.)

✓ by experiment

$$\begin{aligned} I &= Mr \left[\left(\frac{T}{2\pi} \right)^2 g - r \right] \\ &= \frac{1.203}{2} \cdot \frac{2.625}{2} \left[\left(\frac{60/107}{2\pi} \right)^2 \cdot 9.81 - 0.12 - \frac{2.625}{2} \right] \\ &= \end{aligned}$$

② The pendulum method—for axis

“ I ” can be determined by mounting the part on a platform made of light weight platform and is suspended by chords.

Observe the number of small oscillations over a period of time.



Mass property of rigid body (cont.)

M_P : mass of the part

M_t : mass of the table

r : distance from O to the part's C.G.

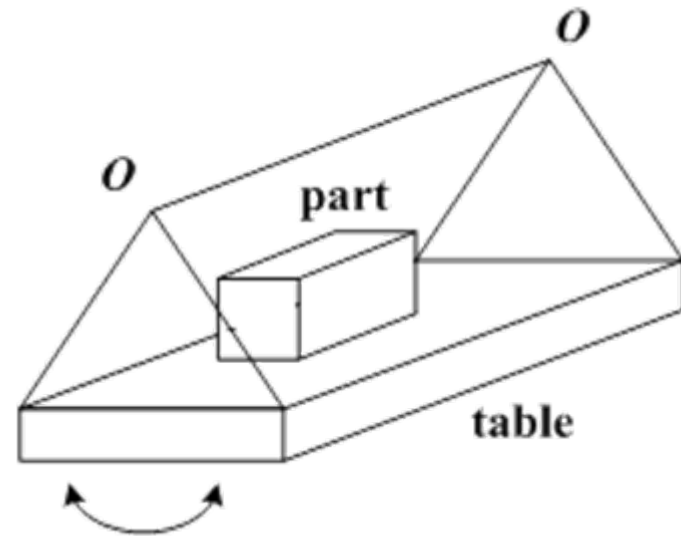
R : distance from O to the table C.G.

I_{Po} : moment of inertia of the part about $O-O$

I_{to} : moment of inertia of the table about $O-O$

T : period of the table + part

T_t : period of the table alone





Mass property of rigid body (cont.)

From previous results :

$$T = 2\pi \sqrt{\frac{I_{Po} + I_{to}}{(M_P + M_t)gr}}$$

r : the distance of O to the C.G. of part+table

$$(M_P + M_t)r = M_P r_P + M_t r_t$$

$$I_{Po} = \qquad \qquad \qquad = I_P + M_P r_P^2$$



Mass property of rigid body (cont.)

$$\therefore I_{to} = M_t g r_t \left(\frac{T_t}{2\pi} \right)^2$$

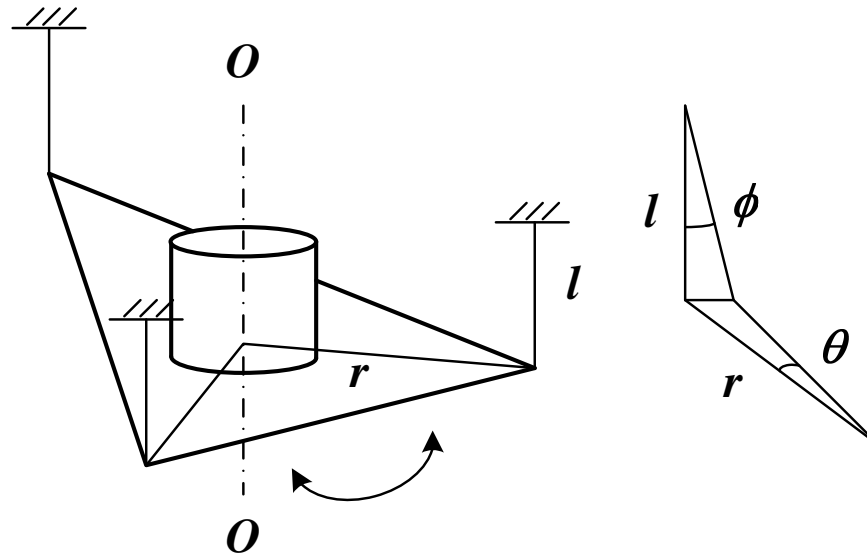
$$\therefore I_P = M_P g r_P \left[\left(\frac{T}{2\pi} \right)^2 - \right] +$$

- ※ The measured quantities:
- ※ It is capable of measuring I whose axis is to $O-O$.
or arrange the desired axis to be to the axis $O-O$.



Mass property of rigid body (cont.)

③ The pendulum method—for axis.



Orient the part on an equilateral triangular (or round) platform such that **its** **axis** is to the three vertical suspension chords, with equal distance r from these chords.



Mass property of rigid body (cont.)

K.E. =

P.E. =

When θ and ϕ are small, P.E. \cong

$$l \sin \phi \approx l\phi = r\theta$$

$$cf : \ddot{\theta} + \frac{Mgr}{I_0} \theta = 0$$

Driving force $\Rightarrow Mgr$ v.s. $\frac{Mgr^2}{l}$

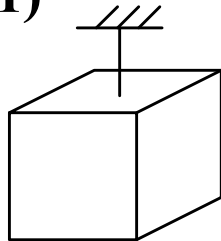
$$\therefore I_{0,p} = \frac{M_p gr^2}{l} \left(\frac{T}{2\pi} \right)^2 +$$



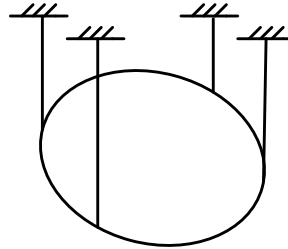
Mass property of rigid body (cont.)

※ Other variants :

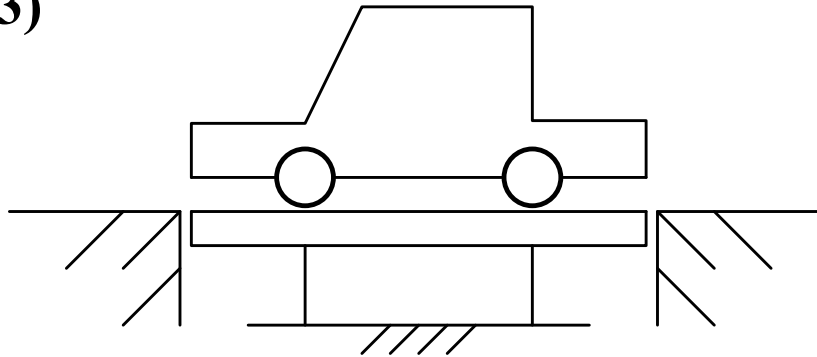
(1)



(2)



(3)



(4) 氣墊台或thrust bearing support

※ 數目不影響系統能力。

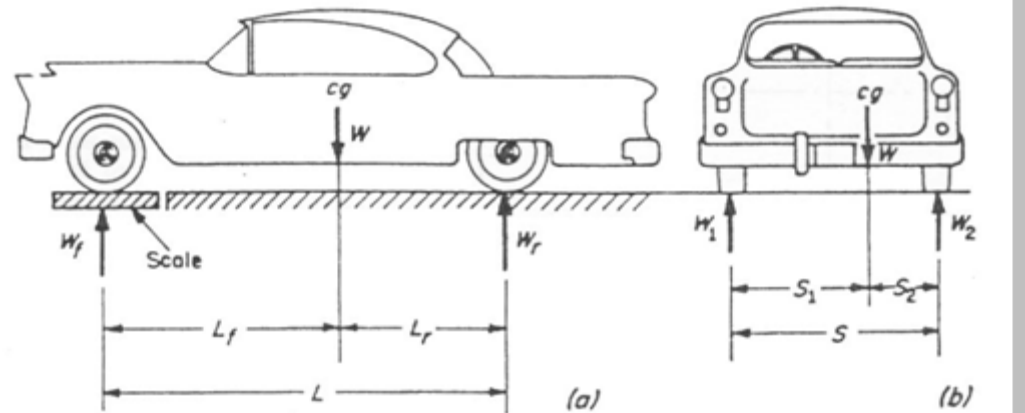


The C.G. of vehicles

- Determination of the C.G. for vehicles.
 - 1) Longitudinal & transverse locations by weighing the vehicle

$$L_f = \frac{LW_r}{W}, \quad L_r = \frac{LW_f}{W} \quad L = L_f + L_r, \quad W = W_f + W_r$$

$$S = S_1 + S_2$$





The C.G. of vehicles (cont.)

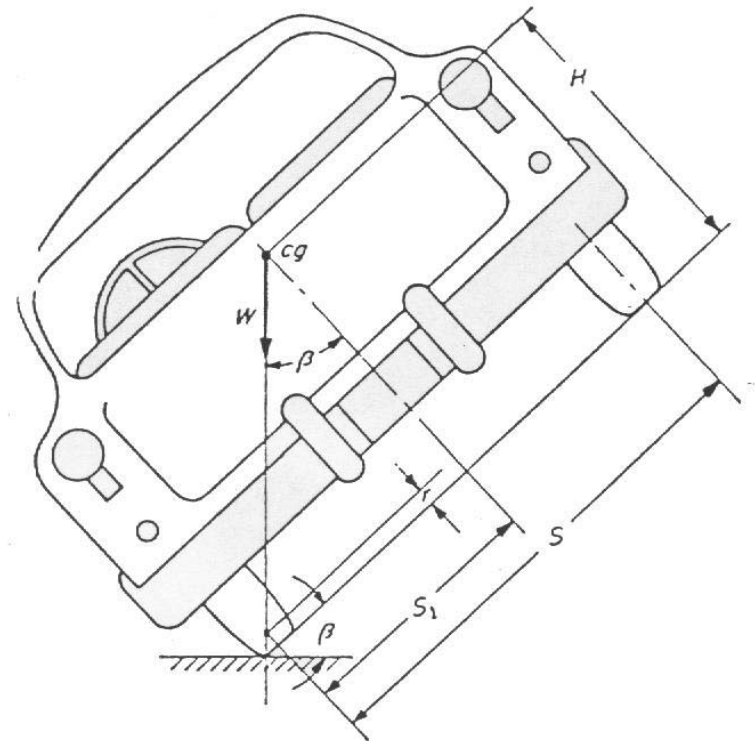
2) Height of the C.G

a) Side tilting

β : angle of inclination,
measured at the
vehicle

r : radius of the tire
cross section at the
contact

S_1 : transverse location of the C.G.





The C.G. of vehicles (cont.)

➤ Comments :

① Known parameters :

② The accuracy depends on θ which is difficult to measure. Its accuracy is

③ Limited to vehicles with front & rear tires

b) One axle elevated

$$H = r_r + h = r_r + \frac{W_f' - W_f}{W} L$$

L : wheel base on level ground

L' : wheel base with rear axle elevated

=



The C.G. of vehicles (cont.)

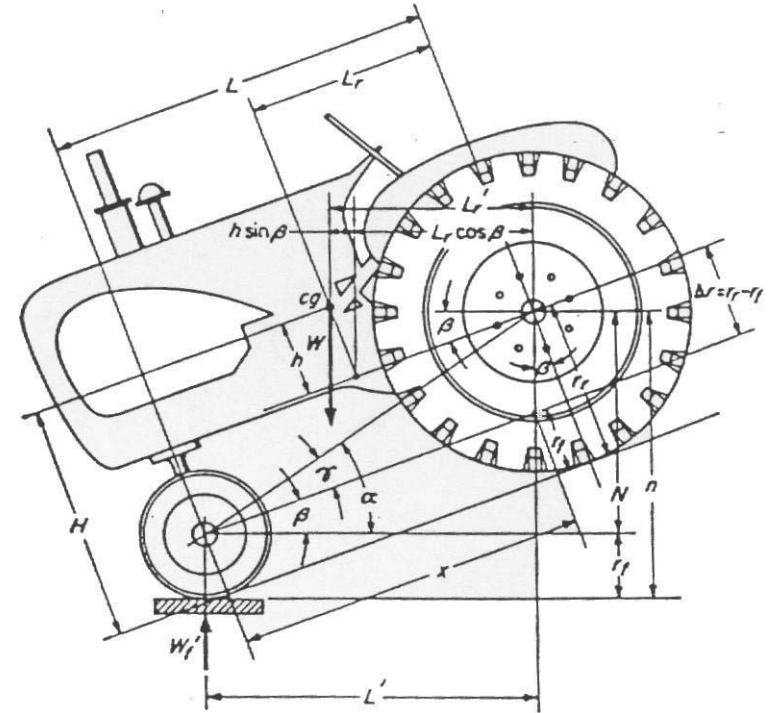
$$\Delta r = r_r - r_f,$$

r_r : rear wheel radius

r_f : front wheel radius

W_f : weight of the front axle on level ground

W_f' : weight of the front axle with rear axle elevated



For equal tire radii,

$$\therefore H = r_r +$$



The C.G. of vehicles (cont.)

➤ comments :

① Measured quantities :

② Can be used **of the wheel arrangement**, e.g. 3-wheel vehicles & unequal front & rear tire radii.

③ Accuracy is better than the method 1 and even better by **experiments on different levels or**

c) **Pendulum method**

pendulum structures with different **&** being carefully suspended on knifed-edges. **locations** of the pendulum & their **periods** are assumed known.



The C.G. of vehicles (cont.)

From previous results :

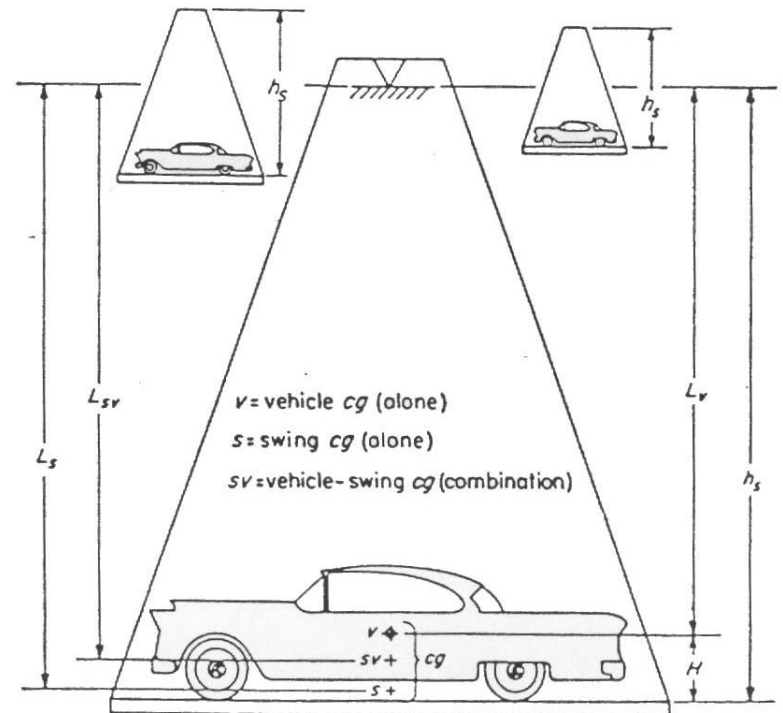
$$I_{P1} = m_v g L_{v1} \left[\left(\frac{T_1}{2\pi} \right)^2 - \frac{L_{v1}}{g} \right] +$$

m_v : vehicle mass.

L_{v1} : from O to the vehicle C.G.

L_{t1} : from O to the table C.G.

: the C.G. height





The C.G. of vehicles (cont.)

The C.G. height can be derived as:

$$H = \frac{A}{B}$$

$$\text{where } A = W_{t1}L_{t1}(T_1^2 - T_{t1}^2) - W_{t2}L_{t2}(T_2^2 - T_{t2}^2) + m_v g (T_1^2 h_1 - T_2^2 h_2) -$$

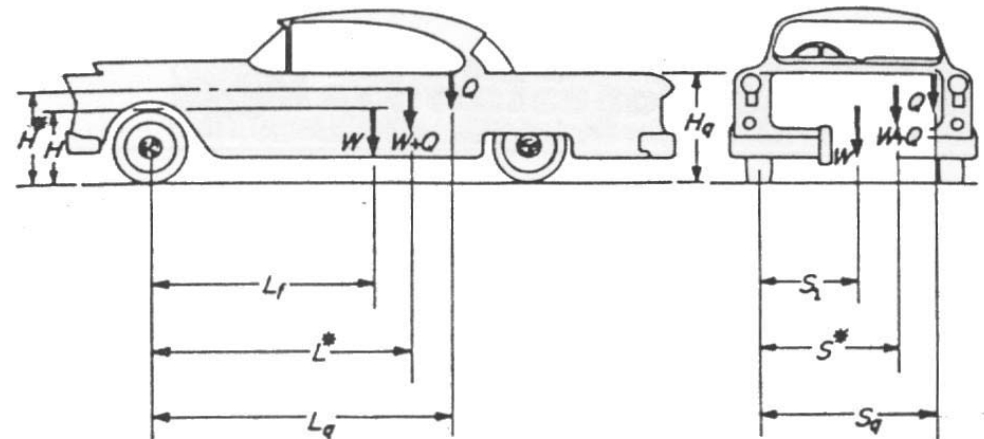
- Comments :
 - ① Measured quantities :
 - ② The accuracy can be within
 - ③ It requires **equipments.**



The C.G. of vehicles (cont.)

3) The C.G. of a loaded vehicle

$$L^* = \frac{WL_f + \sum QL_q}{W + \sum Q},$$
$$S^* = \frac{WS_1 + \sum QS_q}{W + \sum Q}$$



Q : vehicle payload.

→ Especially important on vehicles with payload
its wheel base, e.g. **fork lift truck**,
etc.



END of Mass