Machine Dynamics

Chapter 5a : Mechanical Vibration

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工程評估實驗室

Contents

- Vibration solutions
  - ➢ Problem statements of mechanical vibration
    → Present on the whiteboard
- Fundamentals of vibration theory
  - ➢ Brief review of basic vibration behaviors
    → Present based on the handouts (read it before the class)

Contents (cont.)

- Case study
  - By eliminating vibration
    - ✓ Rigid-rotor balancing
    - ✓ Reciprocating mass balancing
  - By designing system
    - ✓ Translational vibration
    - ✓ Rotational vibration
    - ✓ The whirling effect
- Measurement of mass properties

Case study

- →Rigid-rotor balancing
- →Reciprocating mass balancing
- →Translational vibration
- →Rotational vibration
- →The whirling effect

Rigid-rotor balancing

- Applications －依其 方法
  1) ：
     - ✓ gears, pulleys, cams, flywheels, fans, etc.
  2) ：
     - ✓ engine crankshafts, electric motor armatures, automotive wheels, jet engine rotors, etc.
- Types of rigid-rotor balancing
  1) balance：
     - ✓ The balance of due to the action of . (apply to , minimum balance weight.)

Rigid-rotor balancing (cont.)

2) balance：

- ✓ The balance due to the action of which include & balances. (apply to , minimum balance weights at correction planes in general.)
- ✓ Under special cases, balance weight will be sufficient.

※ The conditions of balance are met

↑ ↓

The conditions of balance are met

Rigid-rotor balancing (cont.)

- Balancing methods

1) Fundamental theory：

$$\left.\begin{array}{l}\sum F = 0 \rightarrow \\ \sum M = 0\end{array}\right\}$$

2) The method 1

$$\sum F = 0$$

$$\overrightarrow{Wr_1} + \overrightarrow{Wr_2} + \overrightarrow{Wr_3} = \vec{R} =$$

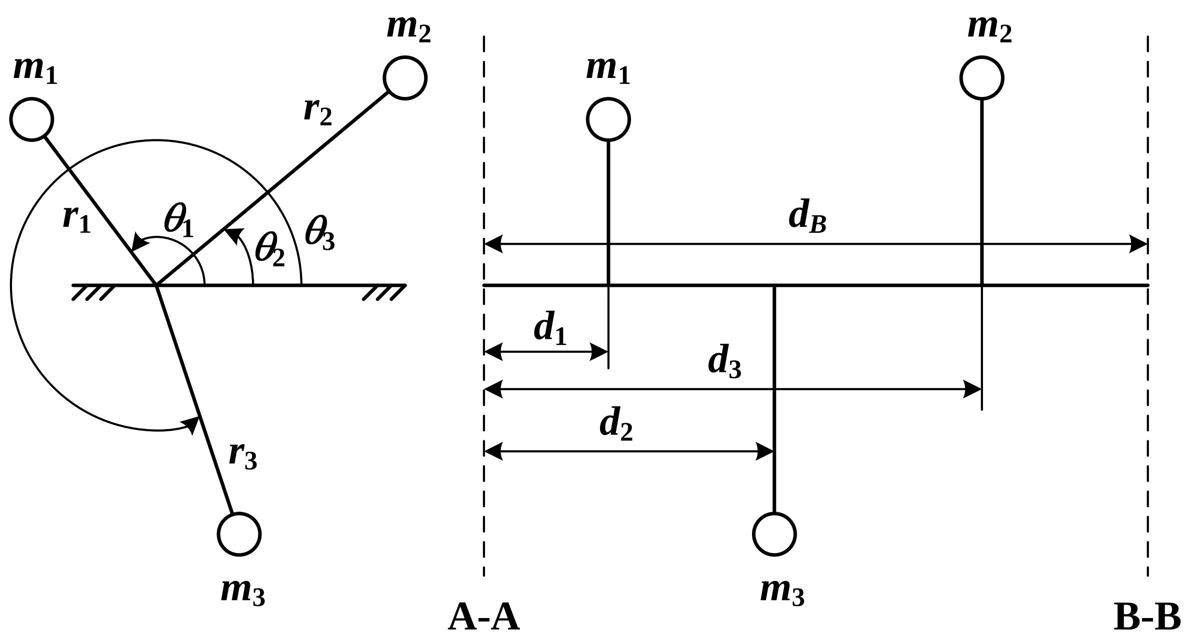
$\rightarrow$ 離心力平衡

Rigid-rotor balancing (cont.)

$\sum M = 0$ → Refer to the plane

$\overrightarrow{Wrd_1} + \overrightarrow{Wrd_2} + \overrightarrow{Wrd_3} = \vec{M}_A =$

→ 離心力矩平衡



Rigid-rotor balancing (cont.)

The two equations can be used to solve for

&

※ 8 parameters：

known conditions by specification：

a) (the positions of correction planes)

b) (the radii of correction masses)

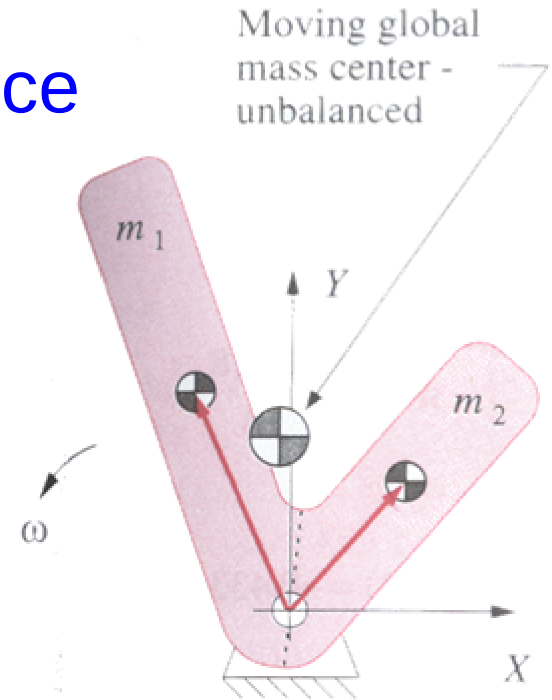
∴ vector equations (scalar equations) can solve for unknowns ( ).

Example

- Thin rotor – plane balance

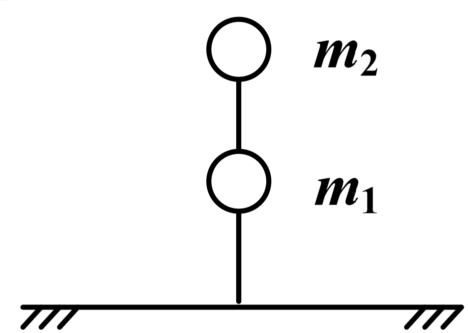
No.	m [kg]	R [m]
1	1.2	1.135@113.4°
2	1.8	0.822@48.8°

$$\omega = 1000\ rpm$$

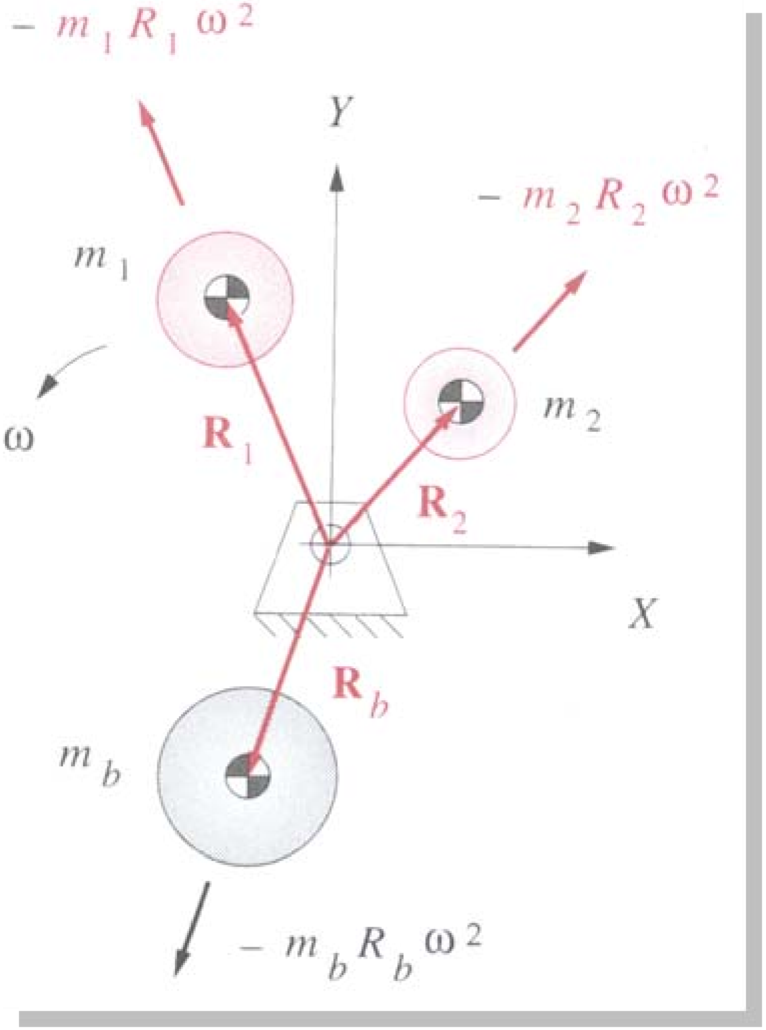


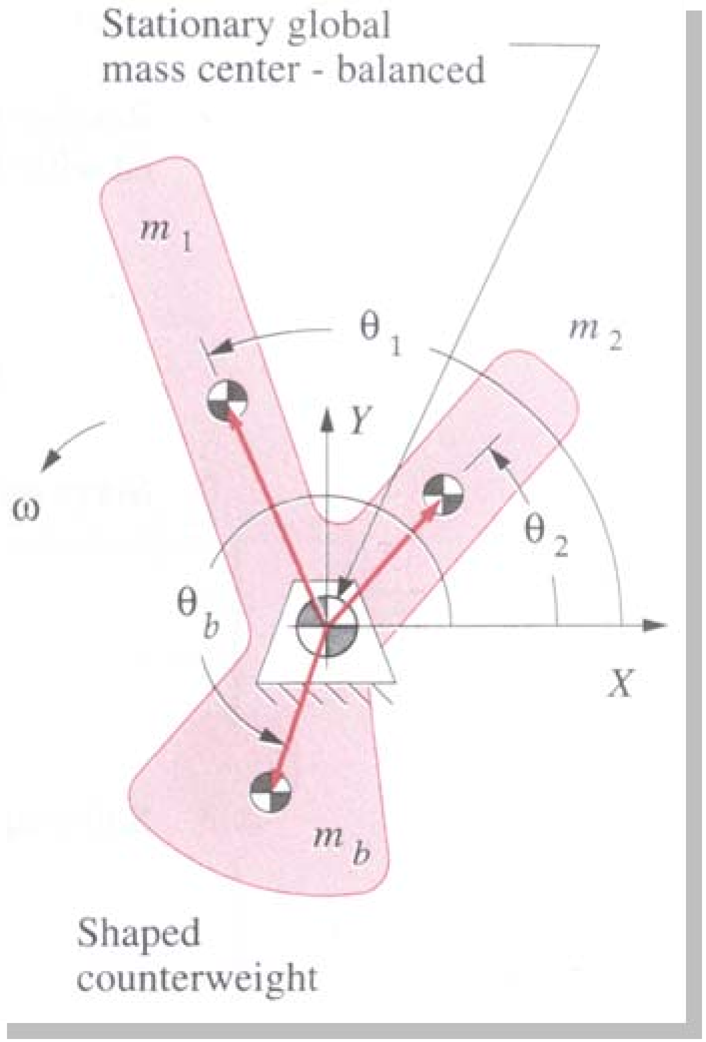
- $R_{1x} =$, $R_{1y} =$
- $m_b R_{bx} =$

  $m_b R_{by} = -2.363$



Example (cont.)





Example (cont.)

$$\therefore \theta_b = \tan^{-1}$$

$$m_b R_b =$$

$$\rightarrow \text{if } R_b = 0.806\ \text{m} \quad m_b =$$

※ Only correction mass is required.

※ Think:
Why is the angular velocity ω irrelevant?

Example (cont.)

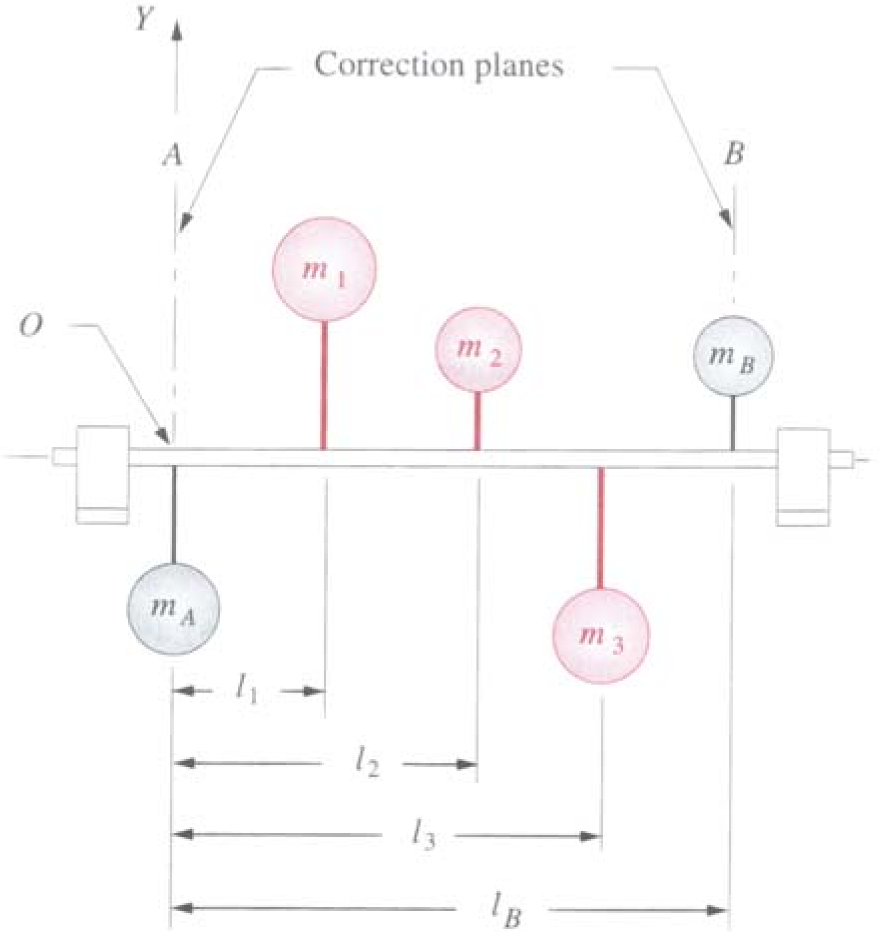
- Long rotor – plane balance

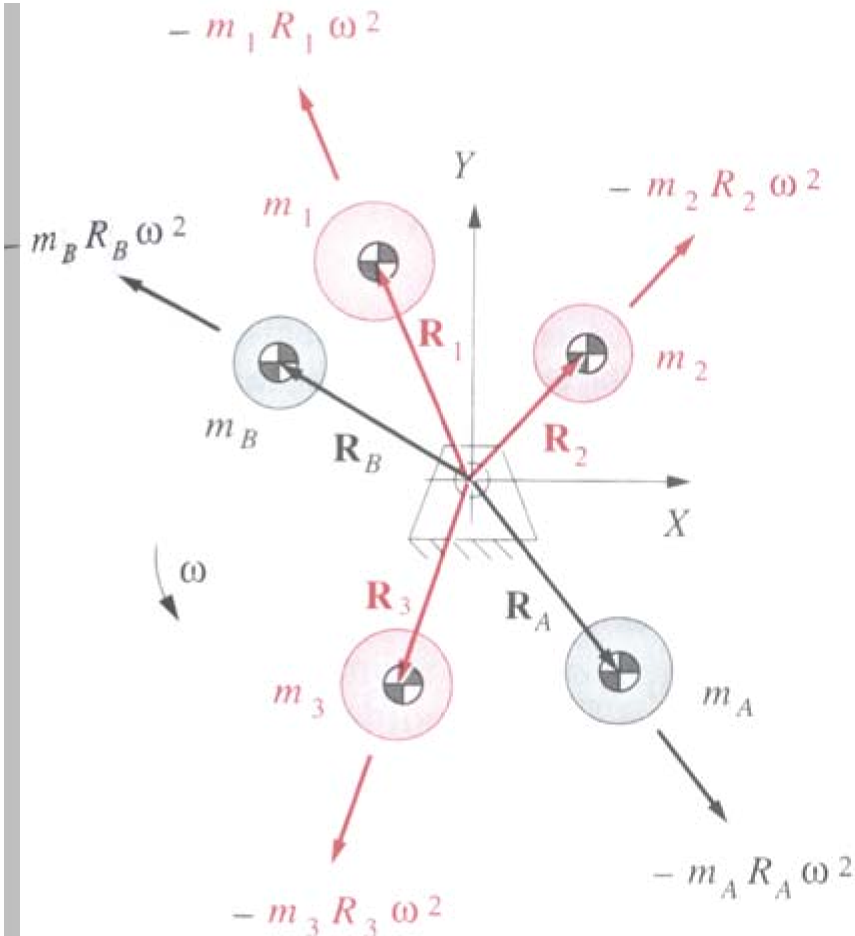
No.	m [kg]	R [m]	distant from plane A
1	1.2	$1.135@113.4^\circ$	0.854
2	1.8	$0.822@48.8^\circ$	1.701
3	2.4	$1.04@251.4^\circ$	2.396
plane B	—	—	3.097

1) Resolve vectors R_i into xy components.

e.g. $\vec{R}_1 = 1.135@113.4^\circ \rightarrow \begin{cases} R_{1x} = \\ R_{1y} = \end{cases}$

Example (cont.)





Example (cont.)

2)

$$m_B R_{Bx} = \frac{-(m_1 R_{1x})l_1 - (m_2 R_{2x})l_2 -}{}$$

$$= 0.230$$

$$m_B R_{By} = 0.874$$

$$\therefore \theta_B = \tan^{-1} \frac{m_B R_{By}}{m_B R_{Bx}} =$$

$$m_B R_B = \sqrt{(m_B R_{Bx})^2 + (m_B R_{By})^2} =$$

Example (cont.)

3)

$$m_A R_{Ax} = -m_1 R_{1x} - m_2 R_{2x} - m_3 R_{3x} - m_B R_{Bx}$$
$$= 0.133$$
$$m_A R_{Ay} =$$
$$= -0.72$$

$$\therefore \theta_A =$$
$$m_A R_A =$$

Rigid-rotor balancing (cont.)

3) The method 2：the approach

a) Balance unbalanced mass on the two correction planes , obtain their resultant forces on these two planes as
.

b) or masses to balance the sum of these resultants on .

✓ Example：

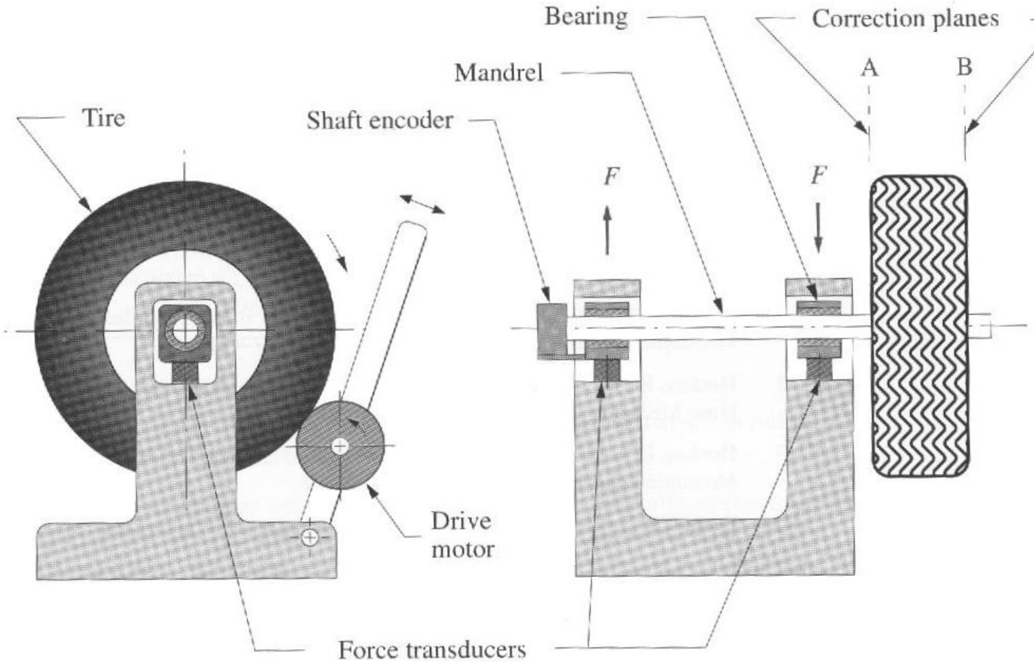
Assume an automotive wheel has unbalanced masses F_1 & F_2 at l_a & l_b, respectively.
Determine the necessary balanced weights at the rim of the wheel.

Rigid-rotor balancing (cont.)

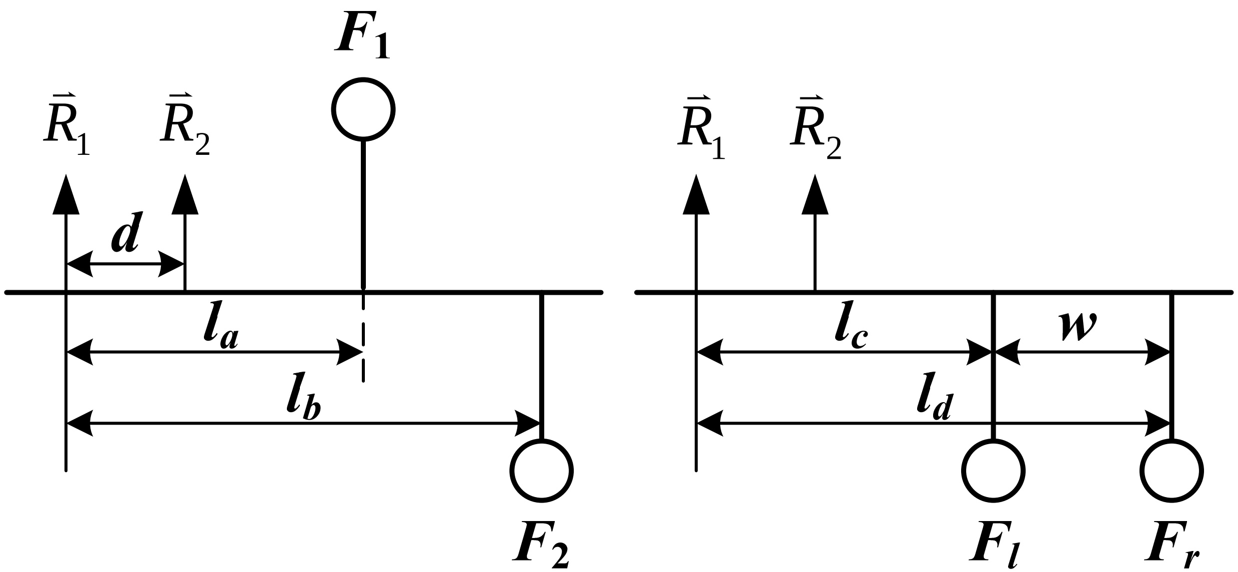
※ The wheel balancing machine

Picture

Schematic diagram



Rigid-rotor balancing (cont.)



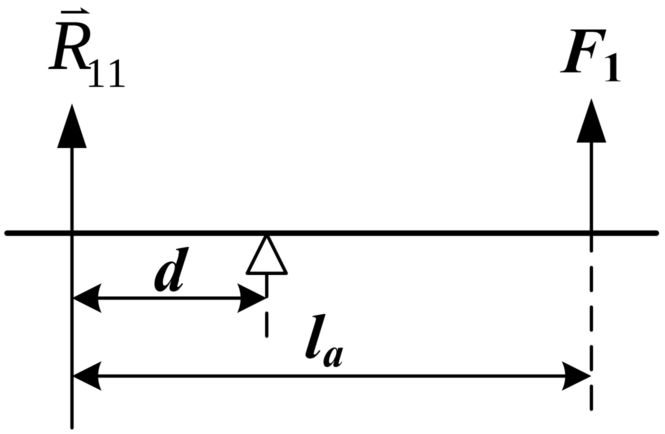
※Key procedures:

- the resultant forces at R_1 & R_2.
- into two respective balance components on l_c & l_d.
- resultants can then be obtained.

Rigid-rotor balancing (cont.)

1) $\left.\begin{array}{l}\vec{F}_1 \rightarrow \vec{R}_{11}, \vec{R}_{21} \\ F_2 \rightarrow \vec{R}_{12}, R_{22}\end{array}\right\} \vec{R}_1, \vec{R}_2 \qquad \left.\begin{array}{l}\vec{R}_1 \rightarrow P_{11}, P_{21} \\ \vec{R}_2 \rightarrow P_{12}, P_{22}\end{array}\right\}$

2) $$\vec{R}_1 = \vec{R}_{11} + \vec{R}_{12}$$
$$= \frac{1}{d}\left[\vec{F}_1\left(l_a - d\right) + \vec{F}_2\left(l_b - d\right)\right]$$
$$\vec{R}_2 =$$



Rigid-rotor balancing (cont.)

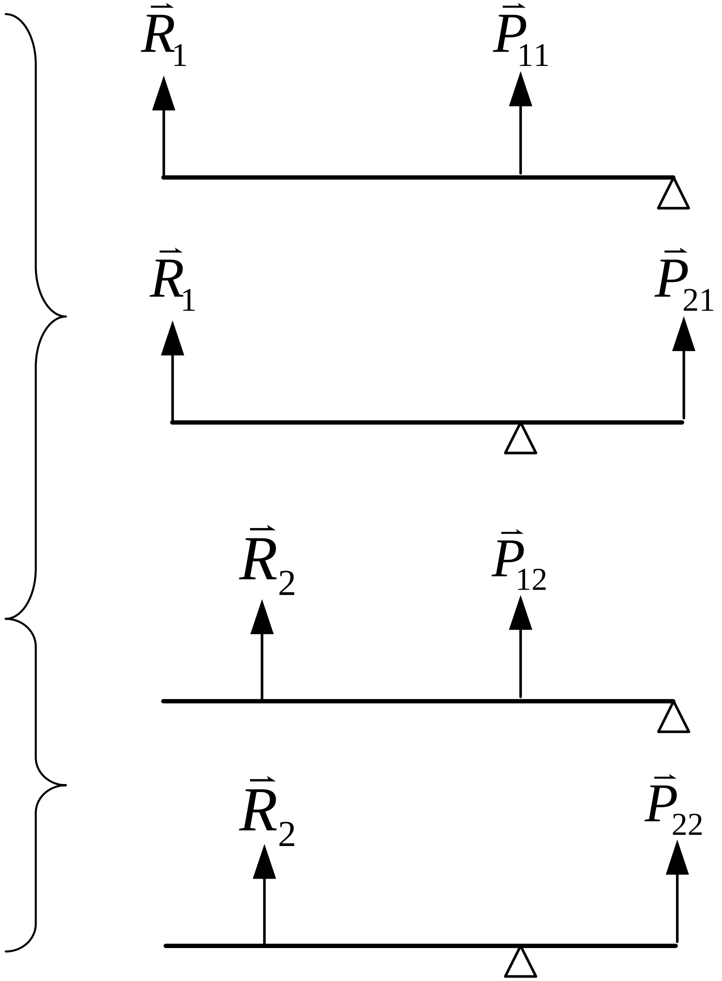
3)

$$P_{11} = \frac{-R_1 l_d}{l_d - l_c} = \frac{-R_1 l_d}{w}$$

$$P_{21} =$$

$$P_{12} = \frac{-R_2}{w}\left(l_d - d\right)$$

$$P_{22} =$$



Rigid-rotor balancing (cont.)

4) $\vec{F}_l = P_{11}e^{i\theta_1} + P_{12}e^{i\theta_2} =$

$\vec{F}_r =$

r_m: the radius of the wheel rim

If R_1 & R_2 are quantities, which should include the effect of , then m_l & m_r are

$m_l =$, $m_r =$

※Think:

Why is the angular velocity ω relevant here?

Comments

- The criterion of application
  - The results can only apply when the rotor is running at less than of its first
.
- Methods of balance

1) mass

e.g. 車輪：容易 、增加系統

2) mass

e.g. 一般情況：不易 取質量、削弱 、減少系統 ；例外： 煞車碟盤。

Comments (cont.)

- The minimum number of mass & its reason

※ For general cases:

The resultant unbalanced moment is in a different axial plane (not in) from the resultant unbalanced .

→ A single mass is to satisfy both requirements.

→ masses are needed in general.

→ Exceptions:

Comments (cont.)

a) For rotors

The moment equilibrium is inherent since the

, will be enough.

→ only balance mass (minimum)

b) For rotors

If all unbalance masses point in the same or opposite directions ( ), the resultant force & moment will direct in difference.

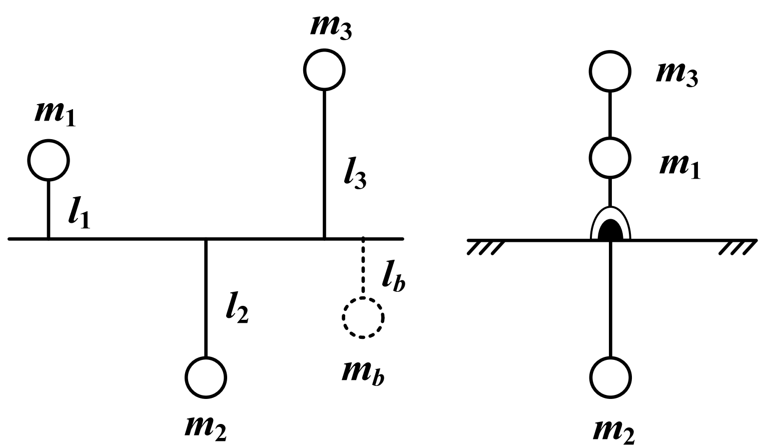
balance mass could be enough.

→ If , mass is enough.

→ If , masses are still needed.

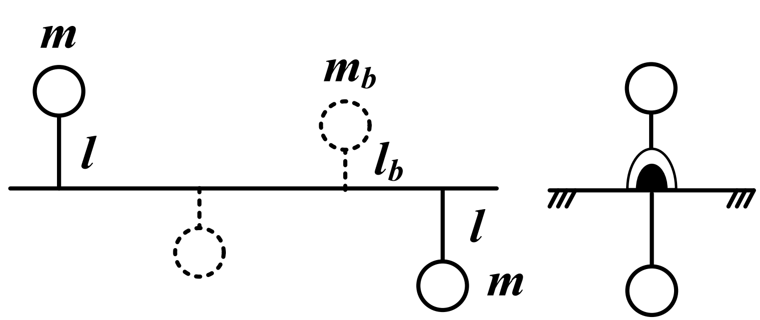
Comments (cont.)

① balance mass



$$\sum F = 0$$
$$\sum M = 0$$

② balance masses



$$\sum F = 0$$
$$\sum M = 0$$

Comments (cont.)

- Minimum vs. more masses

The minimum number of masses, though capable of balancing to the frame, induces on the shaft.

∴ masses can be applied to balance the system. But they will result in:

1) eliminate (advantage)
2) greater total (disadvantage)

Comments (cont.)

e.g. automotive crankshaft: apply a counterbalance weight to unbalance crank.

$\therefore$ The solutions are

- M vs. R in the MR products

1) $R \uparrow \rightarrow M$ but ($\therefore$)

2) The of 2 connection planes $\rightarrow$ product

Note: The system remains for both approaches.

Balancing reciprocating masses

- For a 4-bar linkage

1)

- The resultant of all forces acting on the frame of a machine due to only.
- The frame back & forth.

2)

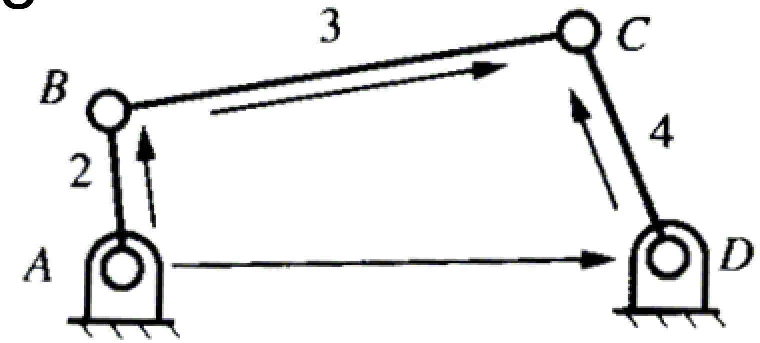
- The moment due to on pivots.
- The frame about the driveline axis.

$\vec{T}_{21}$: input torque, $\vec{F}_{41}$: shaking force on pivot 4

BRM (cont.)

※Complete balance of a machine requires eliminating both the shaking & shaking .

- Balance of a 4-bar linkage
  1) Links are rotating parts which can be balanced as .
  2) Link should be treated differently due to there is center for its motion.



BRM (cont.)

3) In general, reciprocating masses can reach shaking balance by simple means while shaking can only be reduced to a minimum.

- The balancing methods
  1) Method 1 (method)
     shaking force can be balanced by a image of the original mechanism & rotating in the direction.

BRM (cont.)

shaking force can be balanced by its images.

→This method can have balance but is & introduces too much

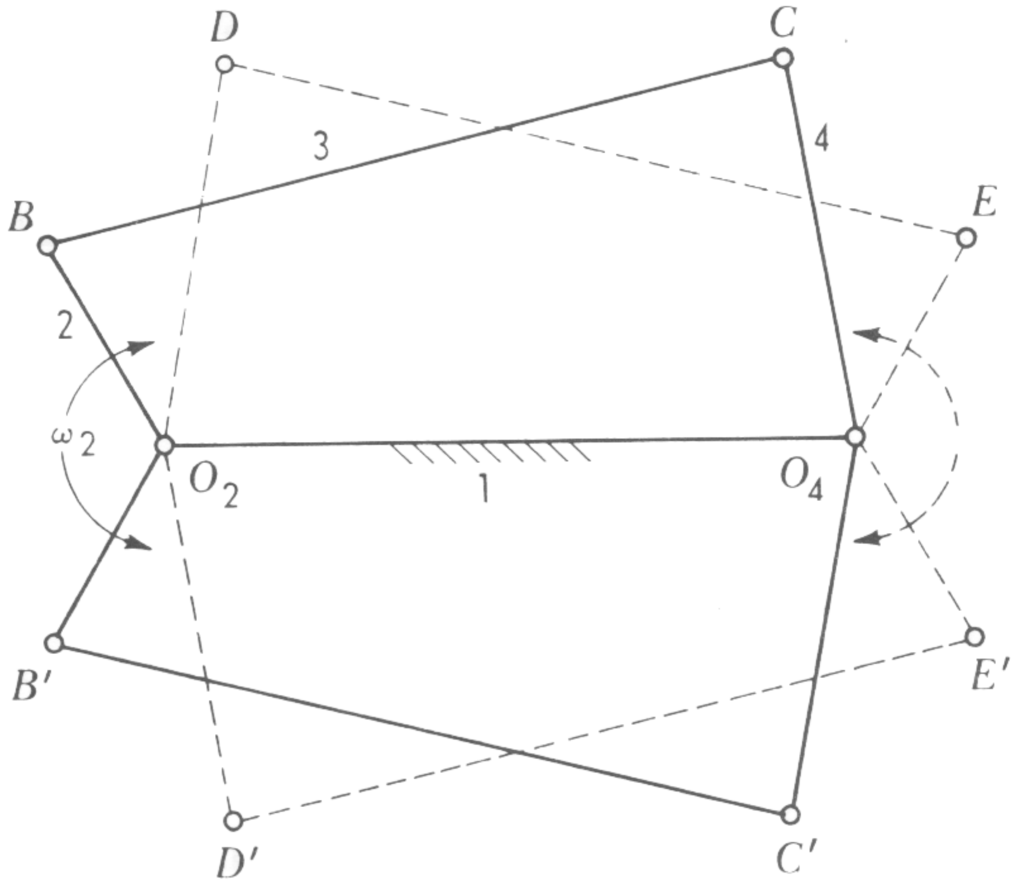
e.g. horizontal opposed engines

→ Not justifiable if just to cancel dynamic effects,

→ Only feasible for production or sharing.

BRM (cont.)

※ shaking force and moment balance



BRM (cont.)

2) Method 2 (method)

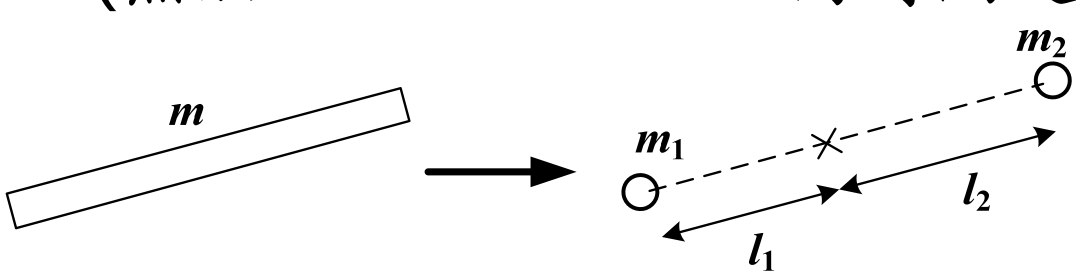
※ For a dynamic equivalent system, it should satisfy：

a)   (same   )

b)   (same   position)

c)   (same   )

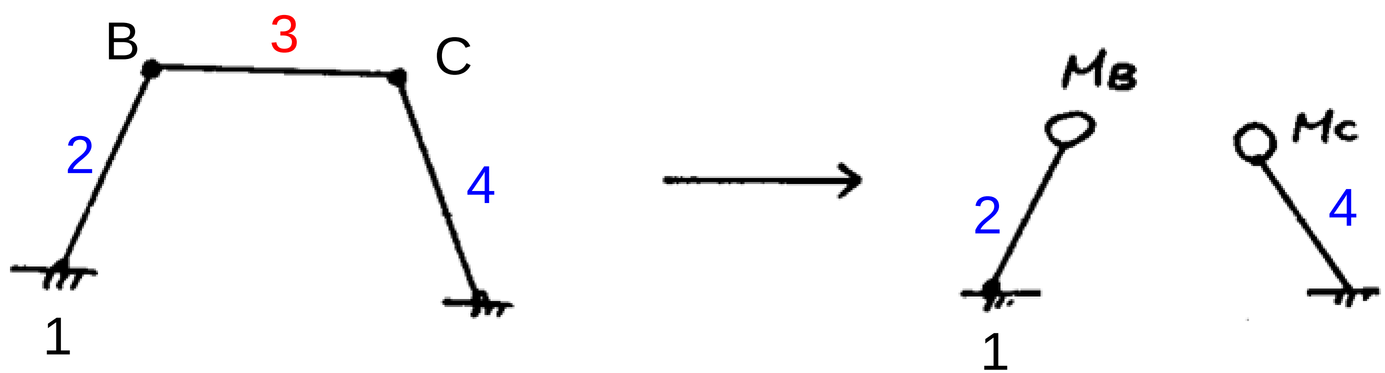
(無法   同時滿足三式)



BRM (cont.)

- Separate the into 2 masses on pivots according to dynamically equivalent equations as follows:

→



BRM (cont.)

- Treating as parts of links 2 & 4.
- Balance these two links separately by the usual way as
- The results:

balance only.

b) Link must be a link.

→ 運用 或 位置，可使link 永遠滿足“ ” link之條件。

BRM (cont.)

3) Method 3 (method)

※ Basic concept:

Shaking force balance

= making the global

= (function of time)

= coefficients of all dependent items equal

※ Still complete balance only

BRM (cont.)

$$m_t = m_2 + m_3 + m_4$$

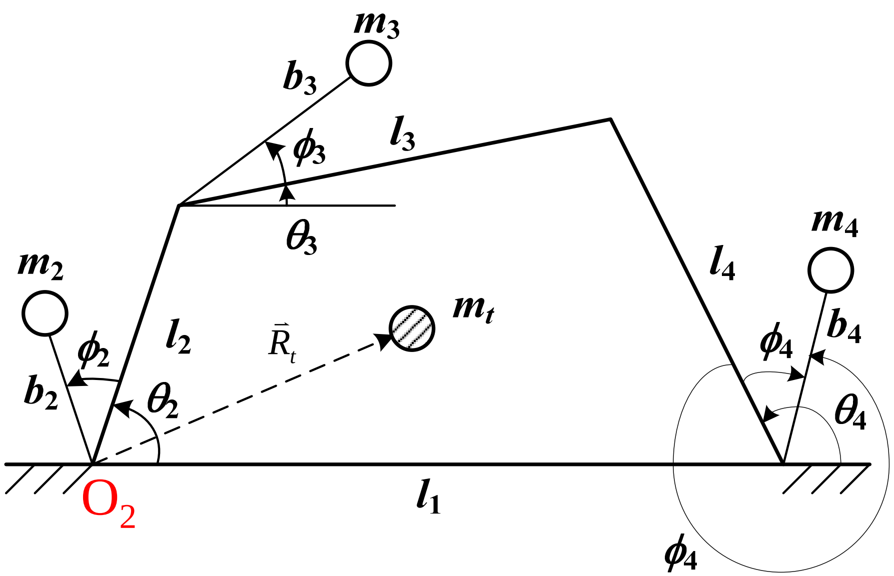
from $\sum M_{O_2} = 0, \quad \vec{R}_t =$

$$\vec{R}_2 =$$

$$\vec{R}_3 =$$

$$\vec{R}_4 = l_1 e^{i\theta_1} + b_4 e^{i(\theta_4+\phi_4)}$$

$$m_t \vec{R}_t = m_4 l_1 e^{i\theta_1} + m_4 b_4 e^{i(\theta_4+\phi_4)} + (\qquad) e^{i\theta_2} +$$



BRM (cont.)

- Time dependent variables are
- In order to let the global stationary, the above equation $m_t\vec{R}_t$ must be independent of
- From the loop closure equation

$$l_2 e^{i\theta_2} + l_3 e^{i\theta_3} - l_4 e^{i\theta_4} - l_1 e^{i\theta_1} = 0$$

$$\rightarrow$$

- Substitute it into the equation of $m_t\vec{R}_t$ to eliminate

BRM (cont.)

$$m_t\vec{R}_t = \left(m_2 b_2 e^{i\phi_2} + m_3 l_2 - m_3 b_3 \frac{l_2}{l_3} e^{i\phi_3} \right) e^{i\theta_2}$$
$$+ \left(m_4 b_4 e^{i\phi_4} + m_3 b_3 \frac{l_4}{l_3} e^{i\phi_3} \right) e^{i\theta_4}$$
$$+ m_4 l_1 e^{i\theta_1} +$$

- For being time dependent, their coefficient must be , so that is

BRM (cont.)

$$\begin{cases} = m_3\left(b_3\dfrac{l_1}{l_3}e^{i\phi_3} - l_2\right) \\ = -m_3 b_3\dfrac{l_4}{l_3}e^{i\phi_3} \end{cases}$$

- If the & location of link 3 are known.
  - → The above equations will render scalar equations
  - → Can solve for unknowns below:

BRM (cont.)

$$\left.\begin{aligned}(m_2 b_2)_x &= m_3\left(b_3 \frac{l_2}{l_3}\right)\\ (m_2 b_2)_y &= m_3\left(b_3 \frac{l_2}{l_3}\right)\\ (m_4 b_4)_x &= -m_3\left(b_3 \frac{l_4}{l_3}\right)\\ (m_4 b_4)_y &= -m_3\left(b_3 \frac{l_4}{l_3}\sin\phi_3\right)\end{aligned}\right\}\ \text{solutions:}$$

Note: m_2 & m_4 is the masses for links 2 & 4.

BRM (cont.)

- Consecutive of force balance

1) Shaking (in general)

→ A good practice is to compromise dynamic force balance with shaking and use other means to reduce vibration .

2) System

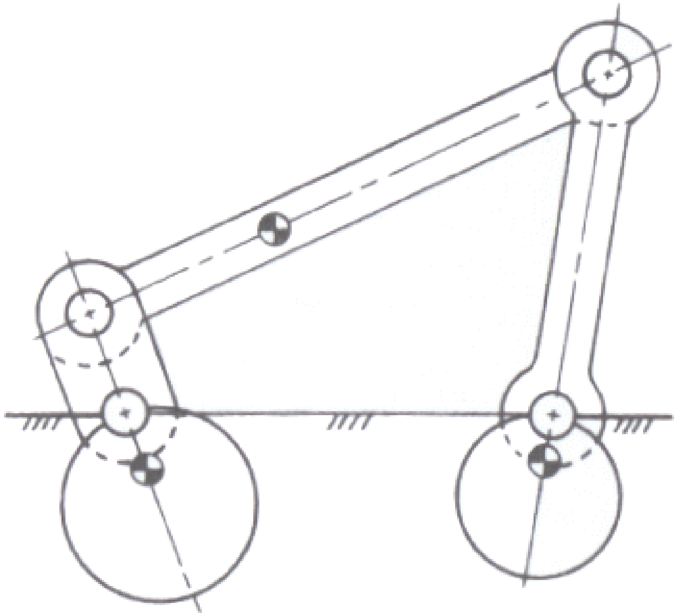
→ Input torque fluctuation

→ Joint force , stress

BRM (cont.)

3) To reduce the effect of on the moment of inertia

→ A shape of counterbalance weight can be applied.



BRM (cont.)

- Practical design considerations of linkage balancing

1) The effect on forces &
2) The effect on the
3) Weight
4)
5) Space
6) Percent of reduction on

END of Chap_5a