Machine Dynamics

Chapter 3a：Classic Dynamics

NTU
MECHANICAL
Department of
Mechanical Engineering
National Taiwan University

Tseng-Ti Fu
Room 503-5
Email: ttfu@ntu.edu.tw
工程評估實驗室

Note

本講義係供上課時補充參考，一切內容仍以上課的講授範圍為準，空缺部份將於課堂中說明。

本段旨在提供 Newton's 2nd law 及 D'Alembert's principle 之假設，理論基礎及其應用方式。

Dynamic systems

- Two categories of machine dynamic systems

A. properties are of the force and couple required to produce the motion

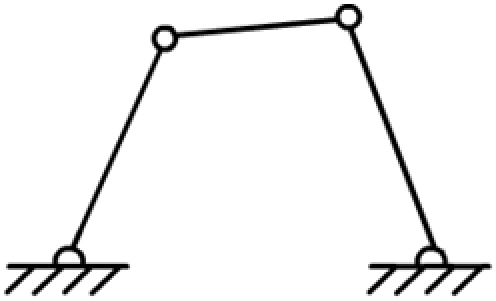
Kinematics →

(acceleration) （free-body diagram）

→

→ Apply equations of equilibrium

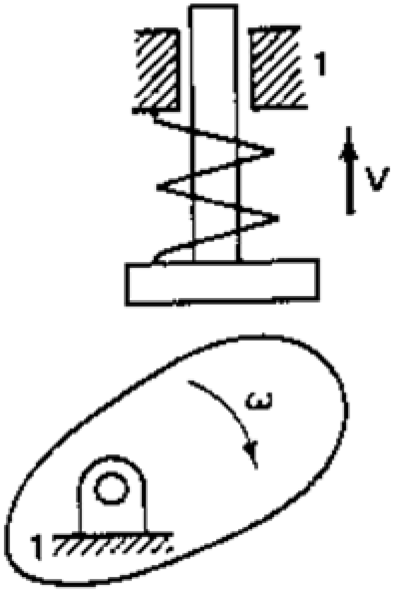
Example: 4 bar linkage



Dynamic systems (cont.)

B. The controls the (mutual dependence)

Example: engine intake & outlet valve systems



kinematics of the follower.

= f (, ,
cam profile, etc.)

→ Apply equations

Type A machine systems

The three approaches

(1) The Newton's second law

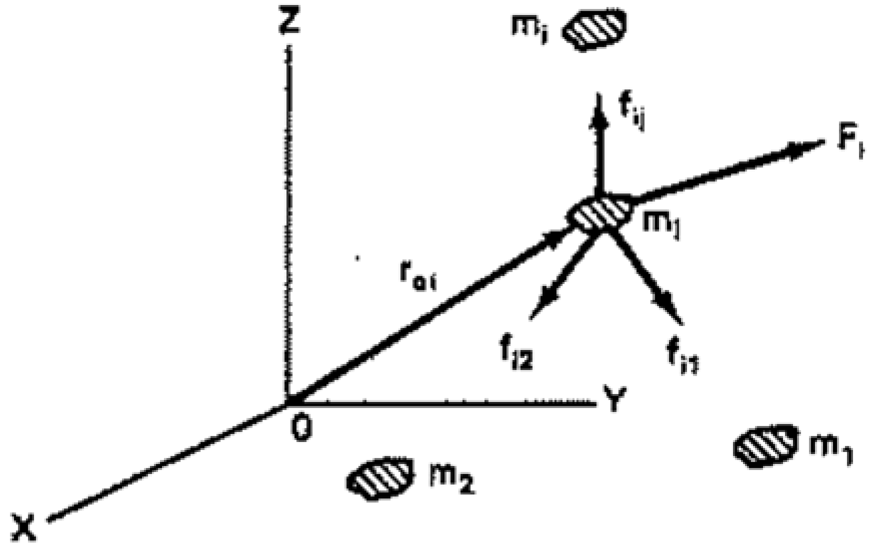
(2) The moment of momentum equation

(3) The d'Alembert principle

Newton's 2nd law

(1) For a single particle of mass m_i subject to external force $\vec{F}_i$ and internal force $\vec{f}_{ij}$.

The Newton's 2nd law is：



where

$\vec{r}_{oi}$ ： the position vector of m_i measured relative to an

Newton's 2nd law (cont.)

$\sum \vec{f}_{ij}$: the resultant force on m_i by all in the system.
In a solid, the summation represents the net effect of all other molecules on the *i*th molecule.

$\vec{F}_i$: is the net effect of all the forces to the system.

(a) :
the force due to direct with other systems of particulars.
e.g. connecting parts in a machine.

(b) :
actions from other systems.
e.g. the gravity

Newton's 2nd law (cont.)

(1.1) extension to the entire system of particles.

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\vec{f}_{ij}+\sum_{i=1}^{N}\vec{F}_i=\sum_{i=1}^{N}\frac{d}{dt}\left(m_i\dot{\vec{r}}_{oi}\right)$$

(1.2) assume the internal forces in a pair of particles are & .

e.g. the force, so the assumption applies to a wide variety of real materials.

$$\therefore \sum_{i=1}^{N}\sum_{j=1}^{N}\vec{f}_{ij}=0$$ ⇨

Newton's 2nd law (cont.)

(1.3) by the concept of the center of mass

$$\vec{R}_g =$$

$\vec{R}_g$: the position vector of the "g"

M : the total mass of all particles

Sum of external forces
= （system mass）×（acceleration of ）

Newton's 2nd law (cont.)

※Note

→For a rigid or nearly rigid solid, the center of mass is in the body.

→Suitable to the study of the motion for rigid & quasi-rigid bodies under 2 conditions：

1) **The mass is**
2) **The internal forces are**

→ the equations of motion but no information about the motion of particles other than

Moment of momentum

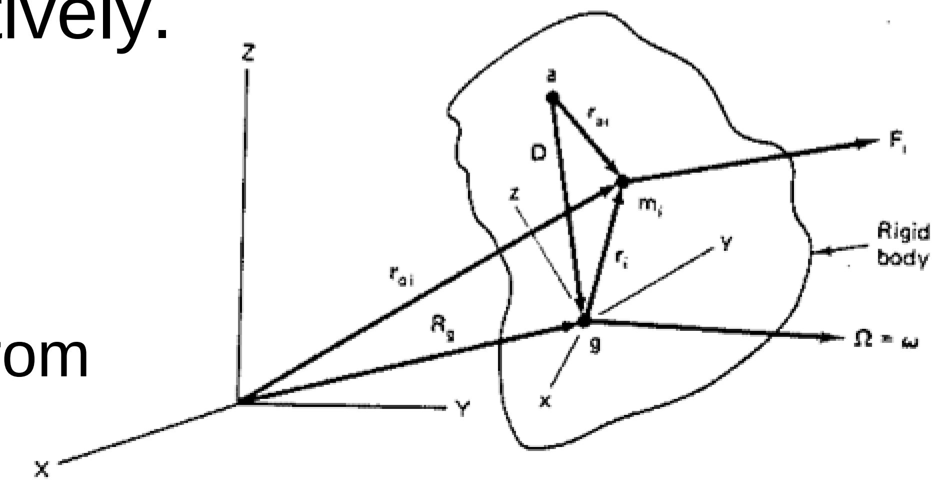
(2) A rigid body with fixed points and a fixed xyz coordinates in the body move together with

$\vec{\Omega} = \vec{\omega}$ and $\dot{\vec{\Omega}} = \dot{\vec{\omega}}$

A typical particle m_i is also in the body and located relative to xyz and an inertia frame XYZ by $\vec{r}_i$ and $\vec{r}_{oi}$, respectively.

$\vec{D}$: the position vector from

$\vec{r}_{ai}$: the position vector from a to the particle m_i



Moment of momentum

$\vec{f}_{ij}$: the forces acting on the mass m_i.

$\vec{F}_i$: the forces acting on the mass m_i.

※Note: This is an alternative form of

∴From Newton's 2nd law, take moment of $\vec{f}_{ij}$ & $\vec{F}_\mathrm{i}$ about an arbitrary point a:

Summing over the whole system

$$\sum_{i=1}^{N} \vec{r}_{ai} \times \vec{F}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \vec{r}_{ai} \times \vec{f}_{ij} = \sum_{i=1}^{N} \vec{r}_{ai} \times \frac{d}{dt}\left(m_i \dot{\vec{r}}_{oi}\right)$$

Moment of momentum (cont.)

(2.1) Assumptions

(1) The internal forces are

$$\vec{r}_{ai} \times \vec{f}_{ij} + \vec{r}_{aj} \times \vec{f}_{ji} = \left(\vec{r}_{ai} - \vec{r}_{aj}\right) \times \vec{f}_{ij}$$

(2) They also joining the particles.

※ A mass in a field experiences attractive forces in this way.
→ conform to a wide variety of real materials.

Moment of momentum (cont.)

(2.2) Expansion

$$\sum_{i=1}^{N} \vec{r}_{ai} \times \frac{d}{dt}\left(m_i \dot{\vec{r}}_{oi}\right) = \sum_i \left(\vec{D} + \vec{r}_i\right) \times \frac{d}{dt}\left(m_i \dot{\vec{R}}_g + m_i \dot{\vec{r}}_i\right)$$

$$= \sum \vec{D} \times \frac{d}{dt}\left(m_i \dot{\vec{R}}_g\right) + \sum \vec{D} \times \frac{d}{dt}\left(m_i \dot{\vec{r}}_i\right)$$

(1) (2)

$$+ \sum \vec{r}_i \times \frac{d}{dt}\left(m_i \dot{\vec{R}}_g\right) + \sum \vec{r}_i \times \frac{d}{dt}\left(m_i \dot{\vec{r}}_i\right)$$

(3) (4)

Moment of momentum (cont.)

(1)式＝

(2)式＝ $\left(\because \sum m_i \dot{\vec{r}}_i = 0\right)$

(3)式＝ $\left(\because \sum m_i \vec{r}_i = 0\right)$

(4)式＝ $\dfrac{d}{dt}\sum(\vec{r}_i \times m_i \dot{\vec{r}}_i) = \sum \dot{\vec{r}}_i \times m_i \dot{\vec{r}}_i + \sum \vec{r}_i \times \dfrac{d}{dt}\left(m_i \dot{\vec{r}}_i\right)$

(first term on the right crossed out with an arrow pointing to 0)

Moment of momentum (cont.)

$$\sum_{i=1}^{N} \vec{r}_{ai} \times \frac{d}{dt}\left(m_i \dot{\vec{r}}_{oi}\right) = \sum_{i=1}^{N} \left(\vec{D} + \vec{r}_i\right) \times \frac{d}{dt}\left(m_i \dot{\vec{R}}_g + m_i \dot{\vec{r}}_i\right)$$

$$\because M\ddot{\vec{R}}_g = \sum_{i=1}^{N} \vec{F}_i$$

$$\dot{\vec{r}}_i = \vec{\omega} \times \vec{r}_i$$

$$\vec{H}_g = \sum_{i=1}^{N} m_i \vec{r}_i \times \left(\vec{\omega} \times \vec{r}_i\right)$$

Moment of momentum (cont.)

∴The dynamic moment equation

∴The moment of about point a

$=$ the moment of the sum of acting at about point a

+ the time rate of about

※ Note:

→ The point a can be an point on the body.

→ All quantities are related to only.

→ Compare to the 3 conditions for

Moment of momentum (cont.)

(2.3) For a continuous rigid body

&

$$\vec{H}_g = \iiint \dot{\vec{r}}_i \times (\vec{\omega} \times \vec{r}_i) dm$$

if $\vec{r}_i = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$

$\frac{d}{dt}\left(\vec{i}\right) = \vec{\omega} \times \vec{i}$, etc.

Moment of momentum (cont.)

(2.4) For xyz system along with the coordinates.

$$\dot{\vec{H}}_g = \vec{i}[I_{xx}\dot{\omega}_x - \omega_y\omega_z(I_{yy} - I_{zz})]$$
$$+ \vec{j}[I_{yy}\dot{\omega}_y - \omega_z\omega_x(I_{zz} - I_{xx})]$$
$$+ \vec{k}[I_{zz}\dot{\omega}_z - \omega_x\omega_y(I_{xx} - I_{yy})]$$

※ Euler's equations: , etc.

Moment of momentum (cont.)

(2.5) For a motion

※ Think:
What are the conditions for the following formula to apply?

$$\sum M = I\alpha \quad \rightarrow$$

D'Alembert principle

(3) D'Alembert equations of motion

→Make equations be solved as the way of ones.

→For a rigid body, the D'Alembert equations are:

X direction
Y direction

※D'Alembert's terms：

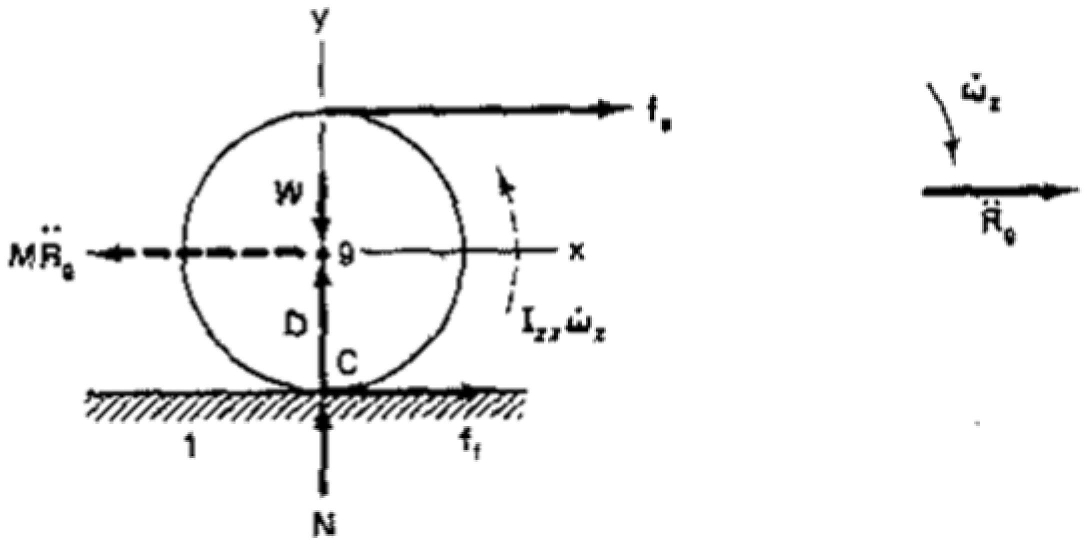
$-M\ddot{\vec{R}}_g$:

$-\vec{D}\times M\ddot{\vec{R}}_g$:

$-\dot{\vec{H}}_g$:

Example

- A cylinder rolls on a plane



(a) Create the   system

(b) Identify the   terms：

(c) Point C為   →

Example (cont.)

(d) The equilibrium equations of motion (EOM)

$x:$

$y:$

$\mathrm{M}_C:$

$\Rightarrow \mathrm{z\ axis}:$

$\mathrm{Kinematics\ of\ rolling}:$

Example (cont.)

※ *unknowns* :

→ equations to solve unknowns.

→

※ The theorem is automatically satisfied.

$$DM\ddot{R}_g + I_{zz}\dot{\omega}_z = D^2M\dot{\omega}_z + I_{zz}\dot{\omega}_z = \underline{\underline{\qquad\qquad}}$$

Physical meaning: transfer from

Example (cont.)

※ In practice, if the condition of is not specified, one may assume it is and calculate

If → The body is

→ The body is , then

→Still equations to solve unknowns.

→

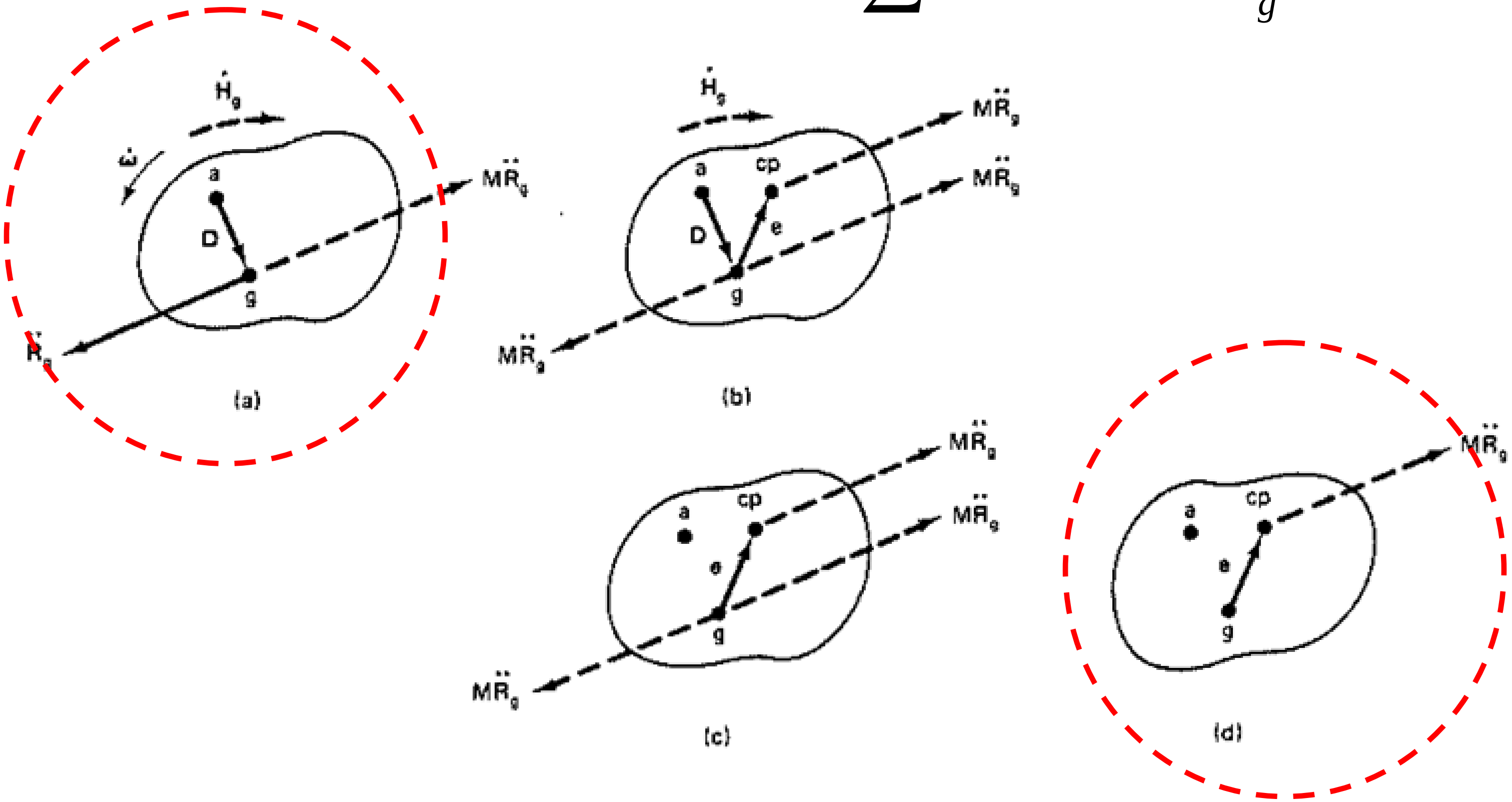
※ The moment equation of motion becomes

The center of percussion
(撞心)

Center of percussion

(a) of free body diagrams

The moment equation is required in the EOM to determine the of $\sum F$ and $M\ddot{\vec{R}}_g$



Center of percussion (cont.)

By resolving into on "g"

1) no net is added. (∵a)

2) no net is added (∵)

They are

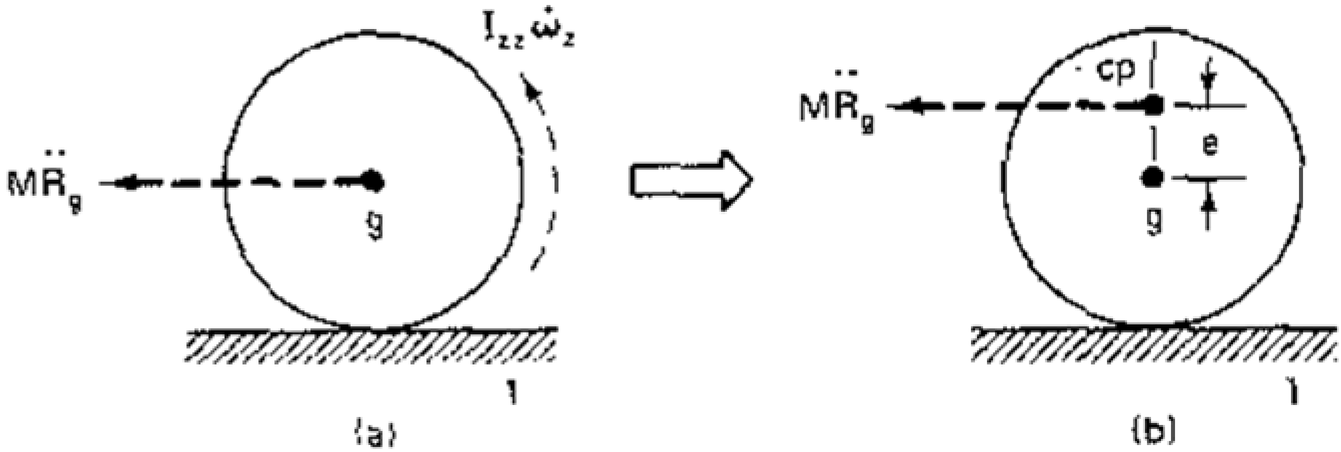
※The point " " is referred as

and can be determined by

(moving from)

Example

- Example:



$$e = \overline{p - g} =$$

In case of rolling→

$$\because I_{zz} = \frac{1}{2} M D^2$$

Example (cont.)

※ For the cylinder, point *p* is a on the cylinder of the velocity of the body

※ In general, the center of percussion is a function of:

(a) the center of , e.g.

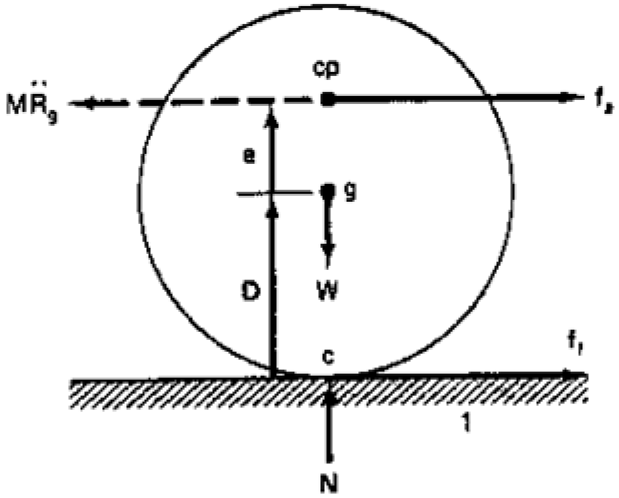
(b) The of the body, in particular, the linear of the cylinder.

e.g.

Example (cont.)

※ If the force f_a is applied at the point p:

$$\left\{ \begin{array}{l} \\ W - N = 0 \\ \\ \end{array} \right.$$



$$\therefore\ f_a = M\ddot{R}_g \qquad W = N$$

Physical meaning:

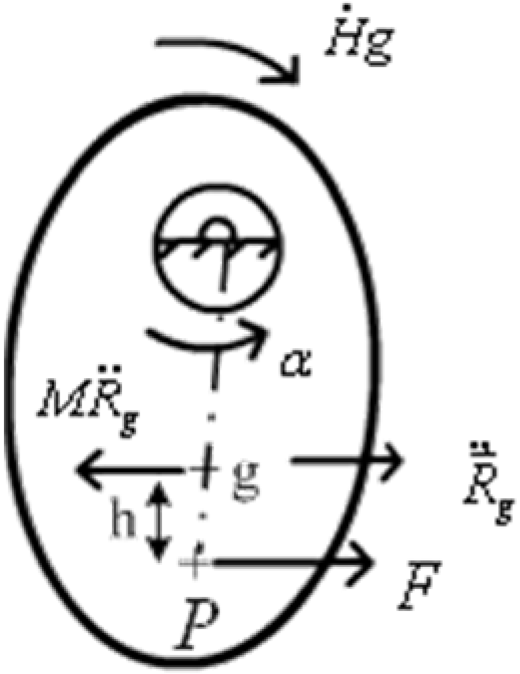
Center of percussion (cont.)

(b) Reduce

if a pendulum has an angular acceleration α

if make

by the moment equation



$$\therefore h = \frac{I\alpha}{F} =$$

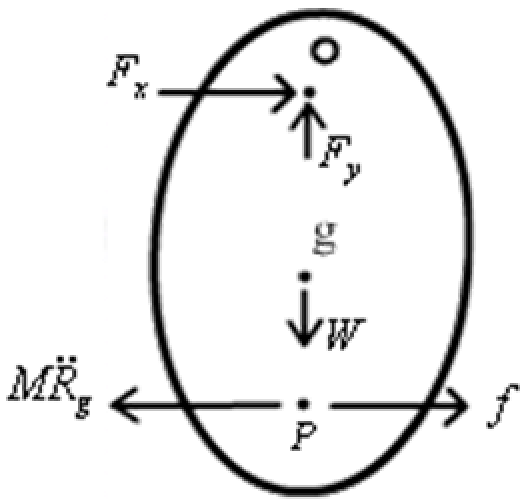
k : radius of gyration

Center of percussion (cont.)

※Therefore, if a force f is applied to the pendulum & hits at its center of

$$f - M\ddot{R}_g + F_x = 0$$

→there will be no at the point of of the pendulum



(外力施於撞心)

※Applications:

1)

2) machine

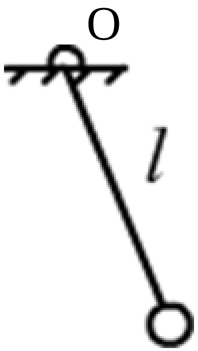
3)

Center of percussion (cont.)

(c) vibration period

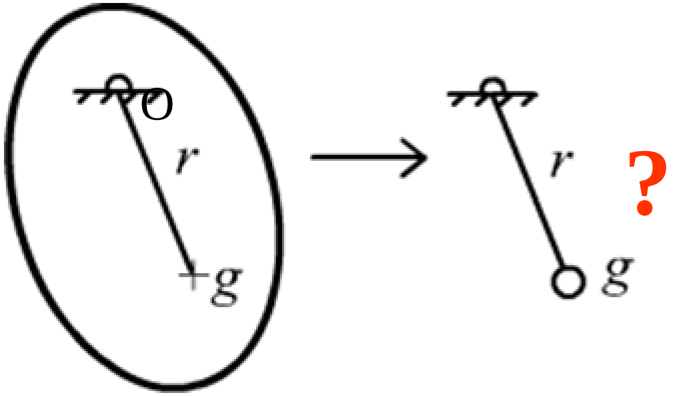
(simple pendulum v.s. compound pendulum)

(1) For a simple pendulum



O：fixed point

(2) For a compound pendulum



Center of percussion (cont.)

→ Derivation:

$$\sum M_0 = I_0 \alpha$$ O：fixed point

if θ is small

$$\ddot{\theta} + \frac{Mgr}{I_0}\theta = 0$$

Center of percussion (cont.)

By applying initial conditions: at $t=0$,

$$\theta=\theta_{\max}\ \&\ \omega=\frac{d\theta}{dt}=0$$

$$\therefore A=0 \quad \& \quad B=\theta_{\max}$$

※Its period:

$$\sqrt{\frac{Mgr}{I_0}}t=2\pi \qquad T=2\pi\sqrt{\frac{I_0}{Mgr}}=$$

k_0 : radius of gyration

Center of percussion (cont.)

※ If all the mass of the compound pendulum are concentrated at , the period of the compound pendulum will be equal to that of the simple one.

※ the center of mass $\rightarrow$ for
the center of percussion $\rightarrow$ for

※ The distance between CG and CP:

$$I_0 = I_g + Mr^2 \quad \rightarrow \qquad Mk_0^{\;2} = Mk^2 + Mr^2$$

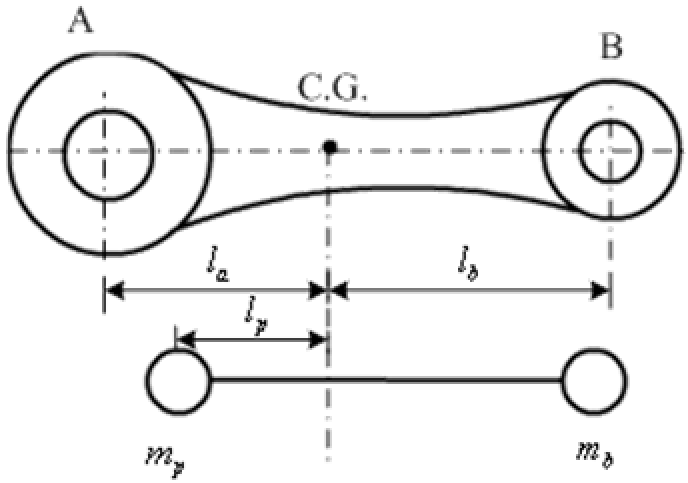
$$k_0^{\;2} = k^2 + r^2 \quad \rightarrow$$

(same as previous *h*)

Center of percussion (cont.)

(d) equivalent system

For a connecting rod, if the mass model is used to simplify it and a two-mass model is used, B is chosen as the rotation center and the dynamically equivalent system must satisfy:



(equivalent)

(equivalent)

(equivalent)

m_c: total mass

Center of percussion (cont.)

$$(\approx \frac{mk^2}{mr} = \frac{k^2}{r})$$

$$m_p = \frac{l_b}{l_p + l_b} m_c \qquad m_b = \frac{l_p}{l_p + l_b} m_c$$

※ where m_p should be put on a distance of from the CG. which is the

※ it may be designed to make so the force at B can be

Center of percussion (cont.)

※ Summary

1) The reaction force will be at the rotation center if the force is applied

2) The are equivalent.

3) are correspondent.

4) The effect of "p" is irrelevant to the

.

5) If $\rightarrow$: the body will move in motion.

END of Chap_3a