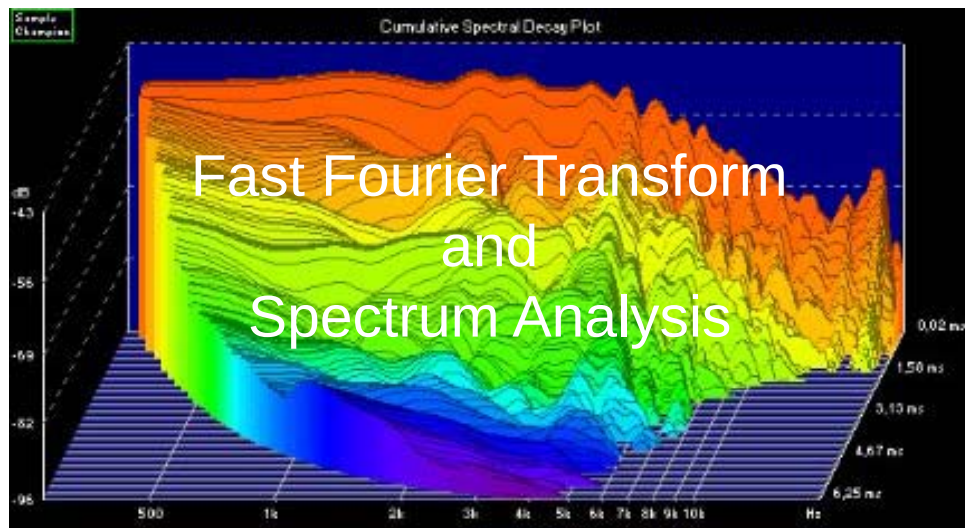




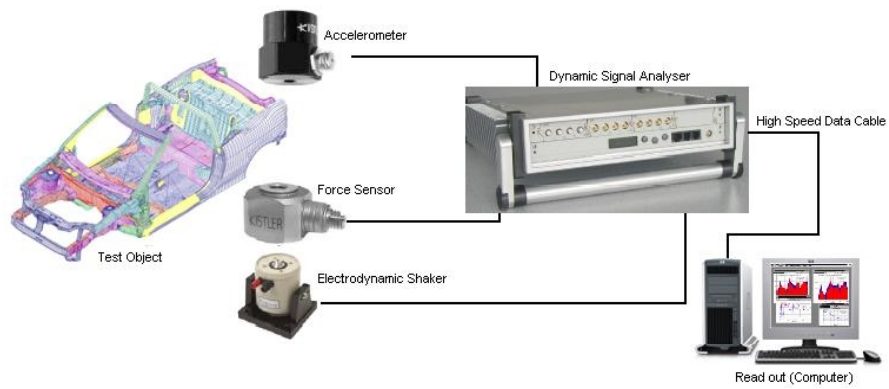
周元昉

Modal testing



周元昉

NTU50235100



http://en.wikipedia.org/wiki/Modal_testing

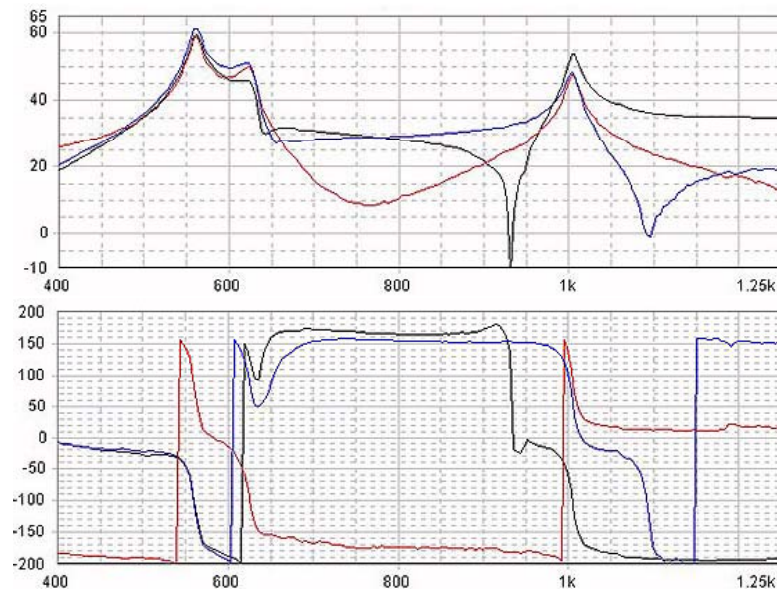
周元昉

NTU50235100



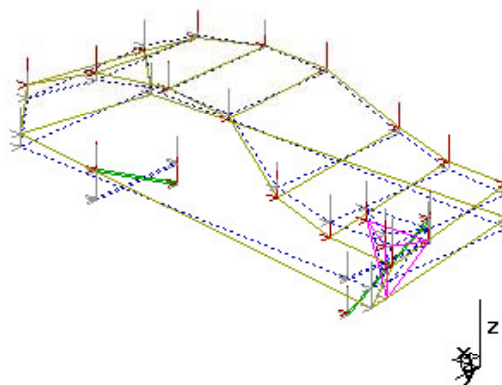
周元昉

Frequency response functions



周元昉

Mode shapes

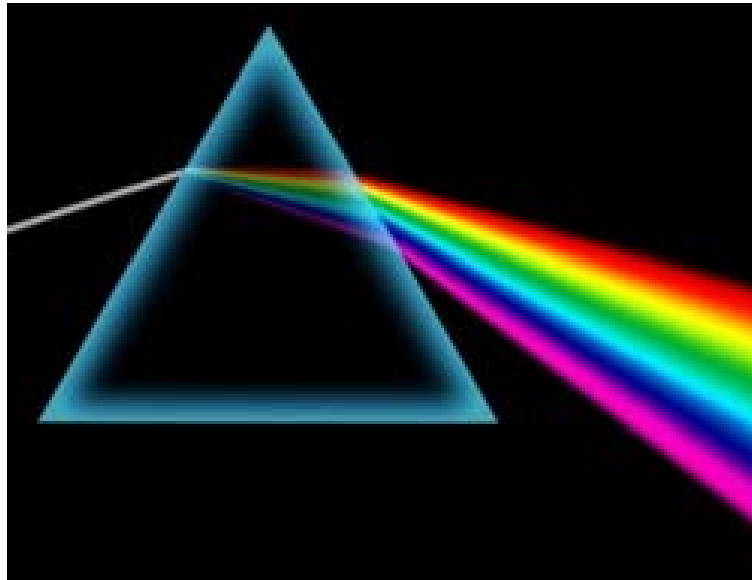


Application:

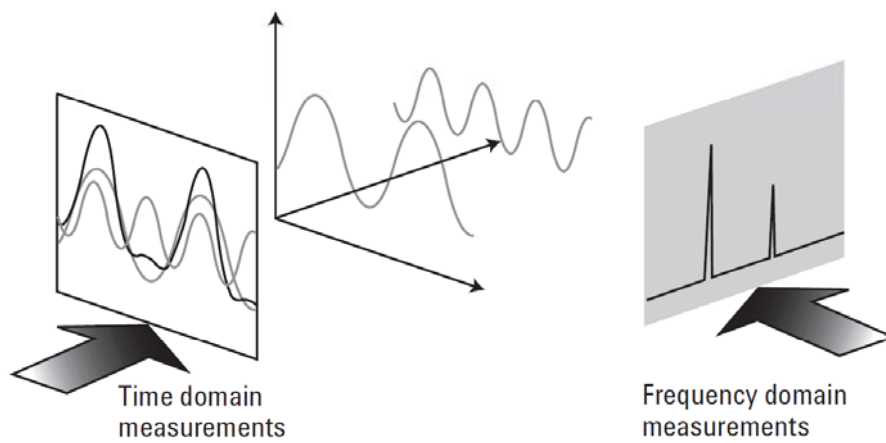
<http://www.youtube.com/watch?v=9kS3dc3n2Y4>

周元昉

Frequency Spectrum



周元昉



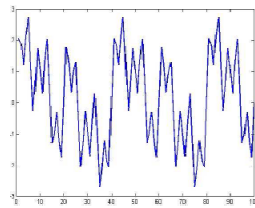
周元昉

Time history of a periodic function --- Fourier series

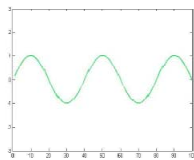


Joseph Fourier (1768 – 1830)

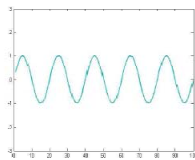
$$y(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$



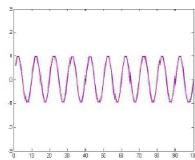
=



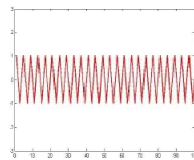
+



+

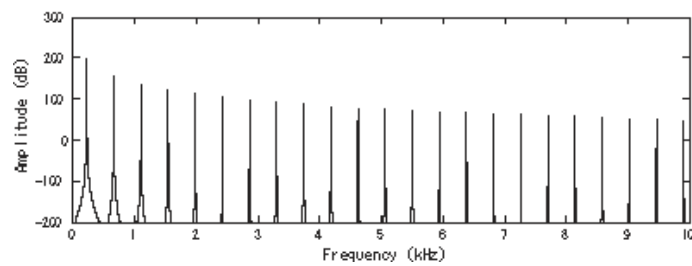
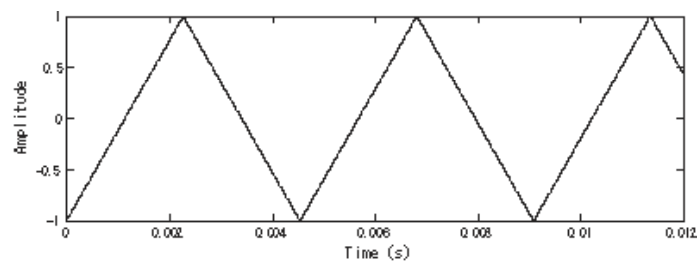


+



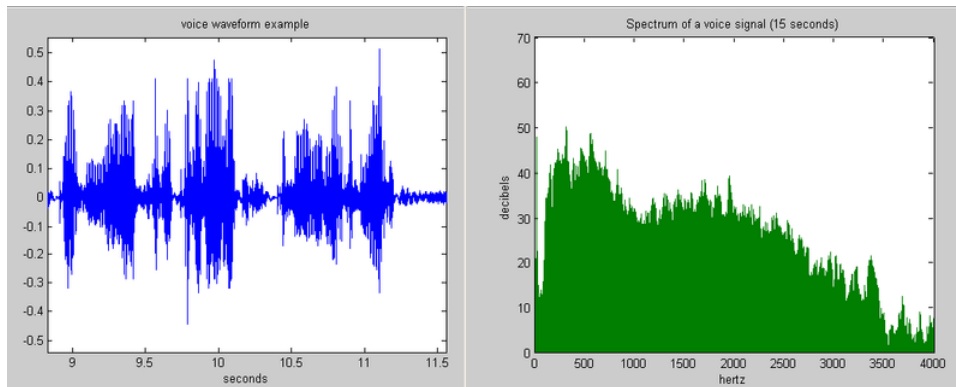
周元昉

Frequency spectrum



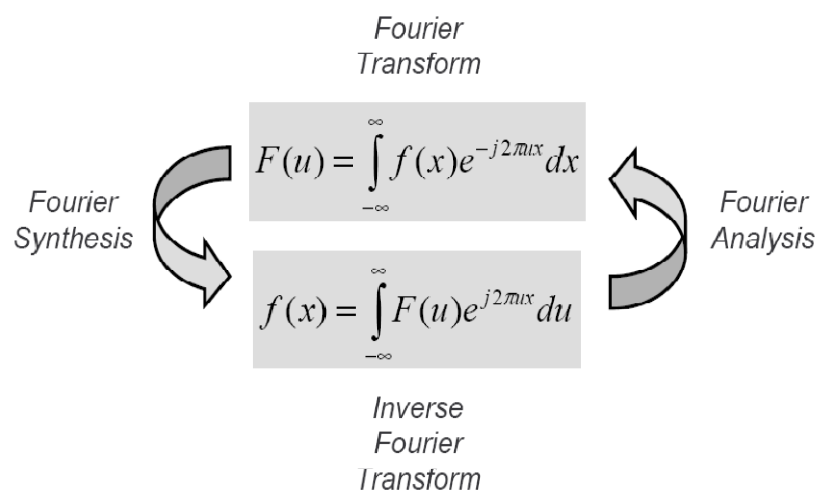
周元昉

Transient signal ---The Fourier transform



周元昉

The Fourier transform



周元昉

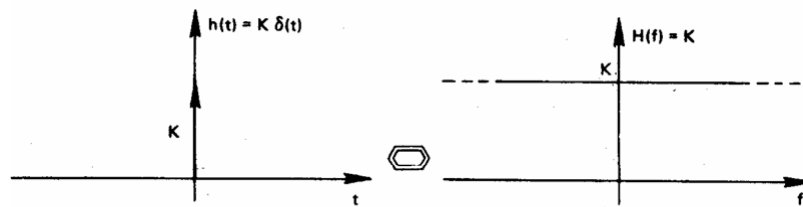


Figure 2-6. Fourier transform of an impulse function.

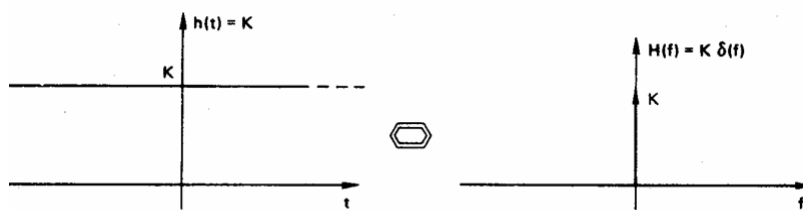
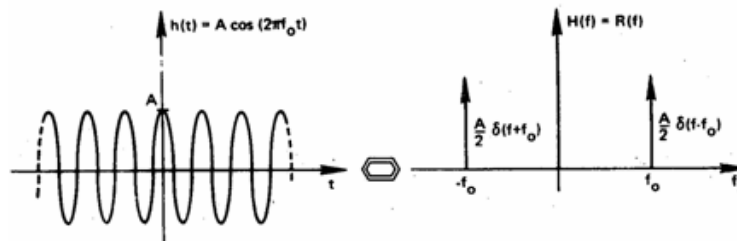
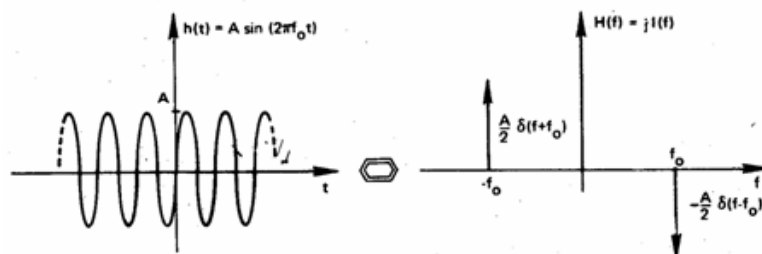


Figure 2-7. Fourier transform of a constant amplitude waveform.

(Ref. E. Oran Brigham, *The Fast Fourier Transform*, Printice-Hall, 1974)

周元昉

Figure 2-8. Fourier transform of $A \cos(at)$.Figure 2-9. Fourier transform of $A \sin(at)$.

周元昉

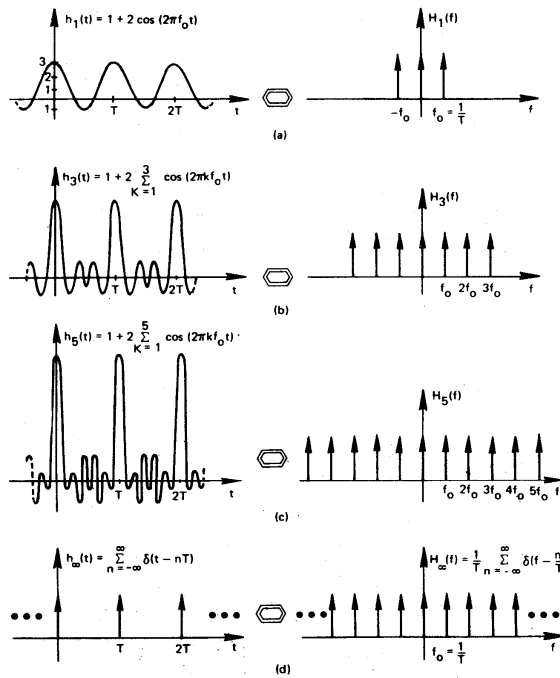


Figure 2-10. Graphical development of the Fourier transform of a sequence of equal distant impulse functions.

周元昉

$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\Updownarrow$$

$$H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$

Fourier transform properties

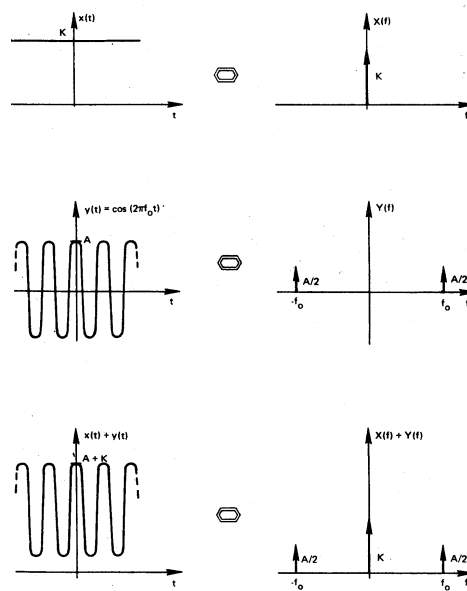


Figure 3-1. The linearity property.

周元昉

NTU50235100

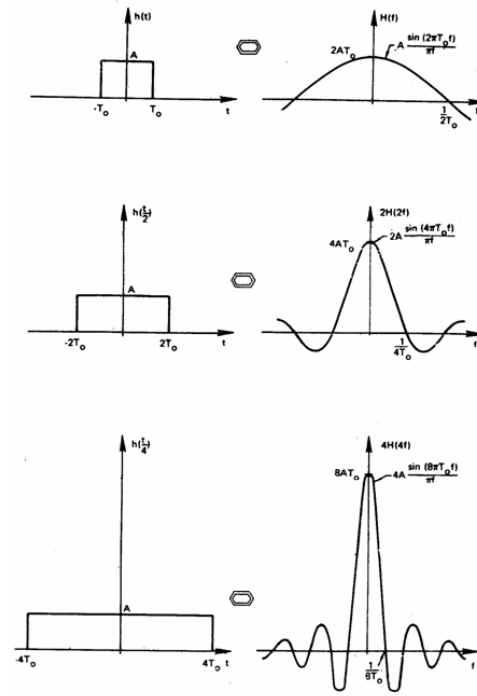


Figure 3-2. Time scaling property.

周元昉

NTU50235100

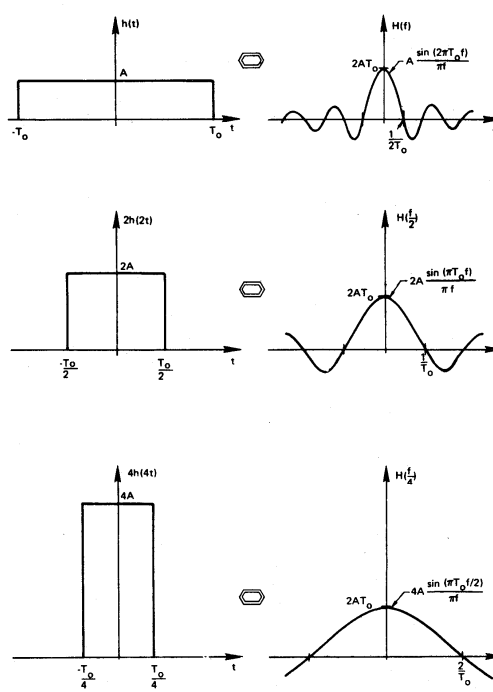


Figure 3-3. Frequency scaling property.

周元昉

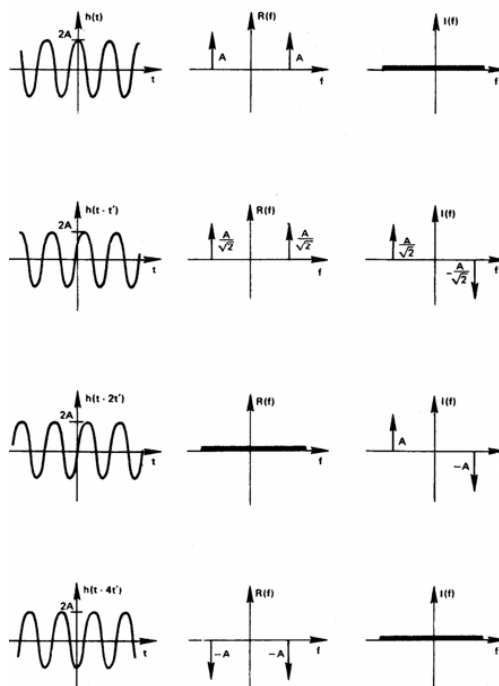


Figure 3-4. Time shifting property.

周元昉

Convolution, Correlation, and Power

Convolution --- time domain

$$[g * h](t) \equiv \int_{-\infty}^{+\infty} g(\tau)h(t-\tau)d\tau \quad \Leftrightarrow \quad G(f)H(f)$$

Correlation

$$\langle g(\tau)h(\tau+t) \rangle \equiv \int_{-\infty}^{+\infty} g(\tau)h(\tau+t)d\tau \quad \Leftrightarrow \quad G(-f)H(f)$$

Autocorrelation if $g = h$

Total power --- Parseval's theorem

$$\int_{-\infty}^{+\infty} |h(t)|^2 dt = \int_{-\infty}^{+\infty} |H(f)|^2 df$$

Autocorrelation is equal to power spectrum $|G(f)|^2$ in frequency domain.

周元昉

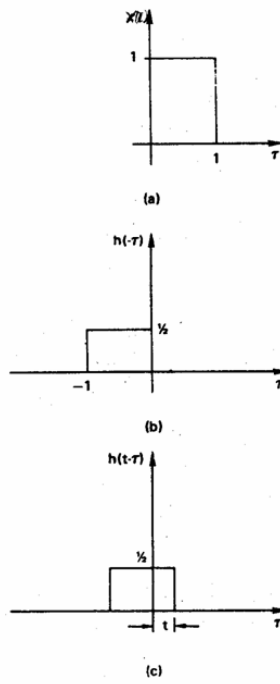


Figure 4-2. Graphical description of folding operation.

周元昉

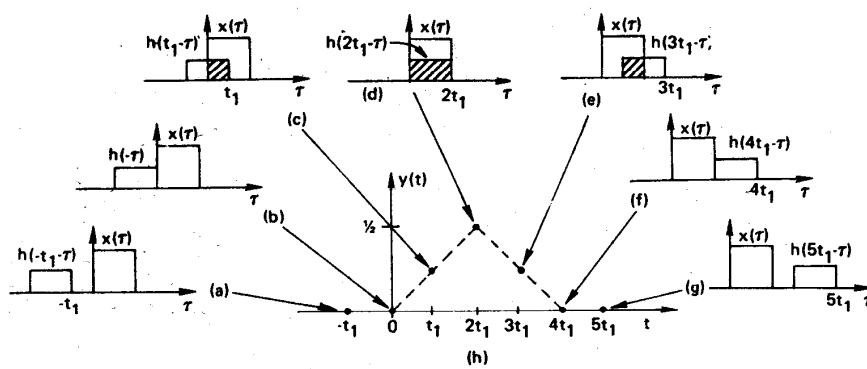
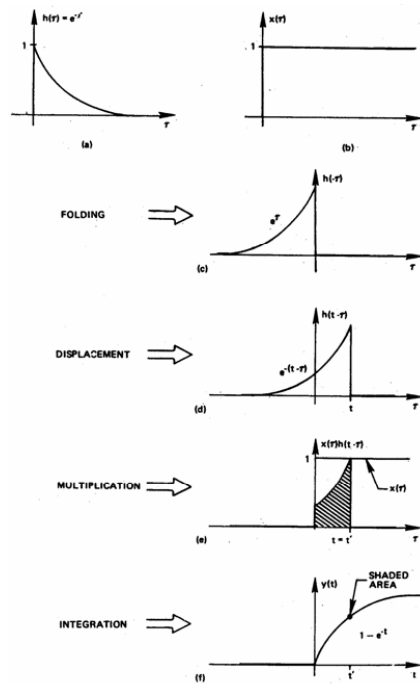


Figure 4-3. Graphical example of convolution.

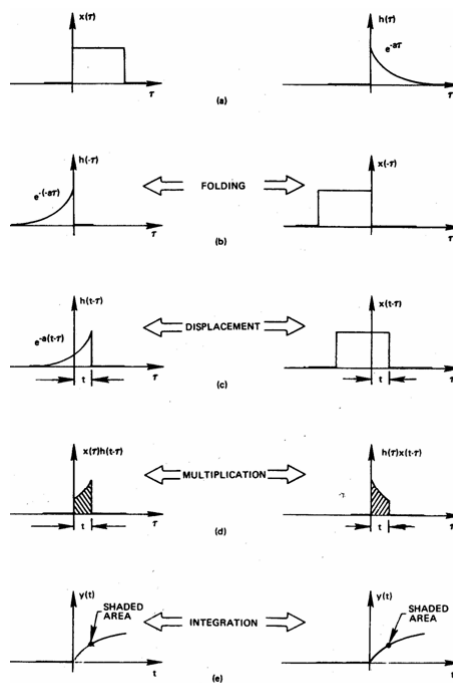
周元昉



NTU50235100

Figure 4-4. Convolution procedure: folding, displacement, multiplication, and integration.

周元昉



NTU50235100

Figure 4-5. Graphical example of convolution by Eqs. (4-1) and (4-3).

周元昉

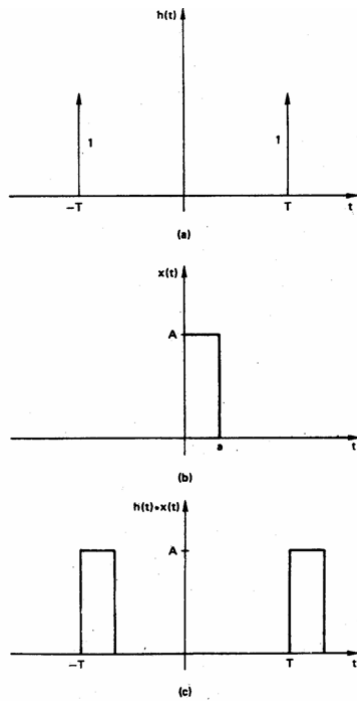


Figure 4-6. Illustration of convolution involving impulse functions.

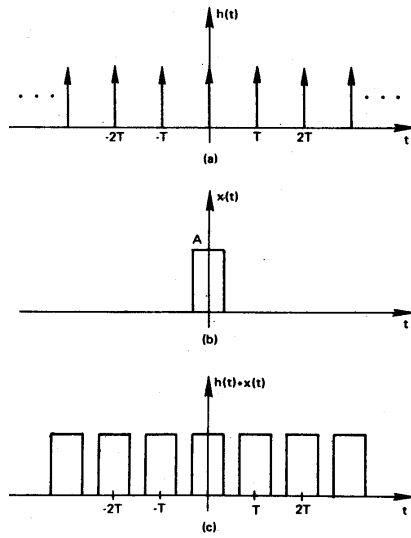


Figure 4-7. Impulse function convolution.

周元昉

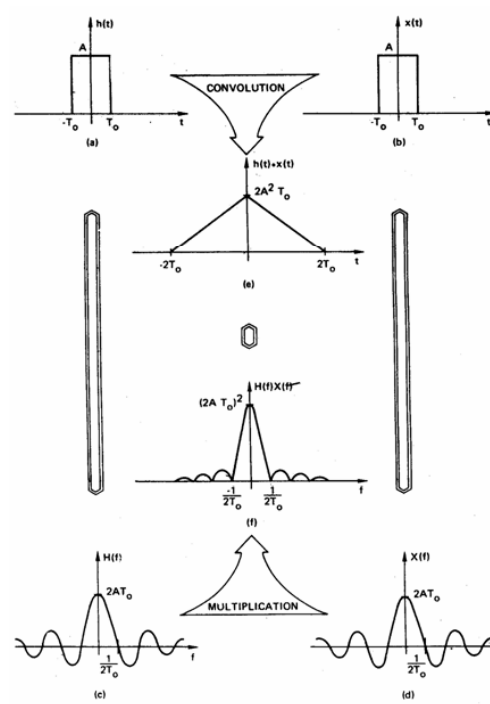


Figure 4-8. Graphical example of the convolution theorem.

周元昉

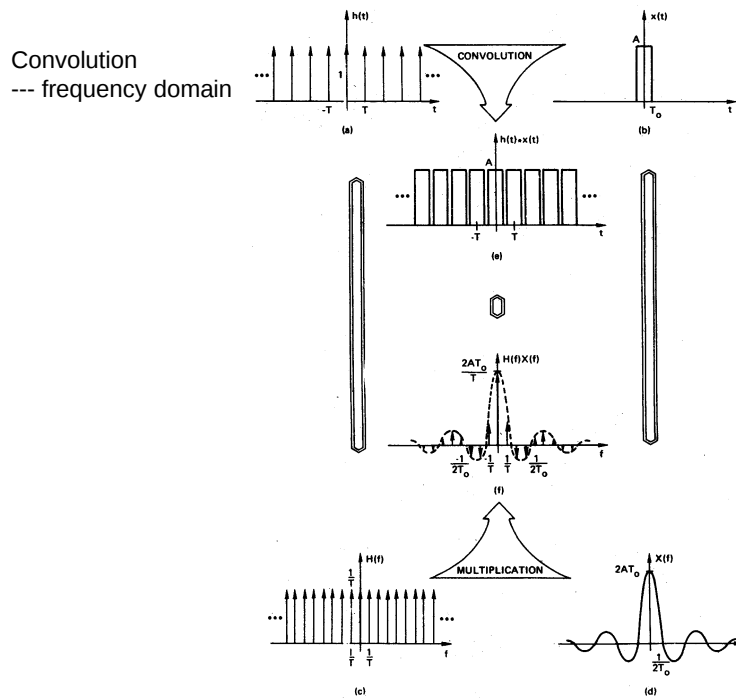


Figure 4-9. Example application of the convolution theorem.

周元昉

Fourier series and sampled waveforms

$$y(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi f_0 t) + B_n \sin(2\pi f_0 t)] = \sum_{n=-\infty}^{\infty} \alpha_n e^{j2\pi f_0 t}$$

$$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad \Leftrightarrow \quad H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$

$$y(t) = h(t) * x(t) \quad \Leftrightarrow \quad Y(f) = X(f)H(f)$$

$$\int_{-\infty}^{\infty} e^{j2\pi ft} df = \delta(t) \quad \int_{-\infty}^{\infty} e^{j2\pi ft} dt = \delta(f)$$

$$h(t)\delta(t - t_0) = h(t_0)\delta(t - t_0)$$

周元昉

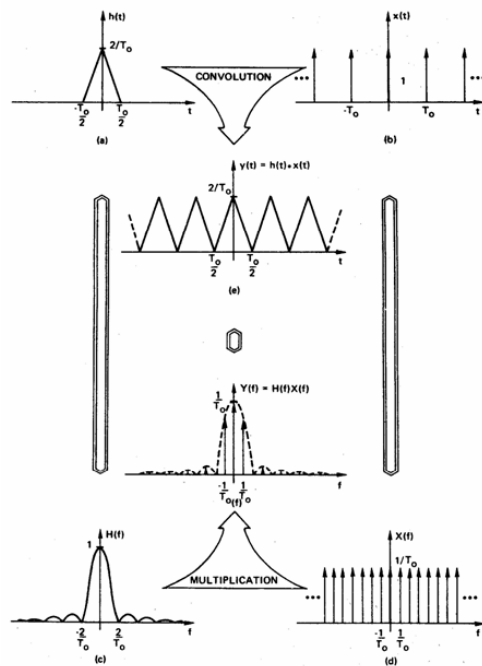


Figure 5-2. Graphical convolution theorem development of the Fourier transform of a periodic triangular waveform.

周元昉

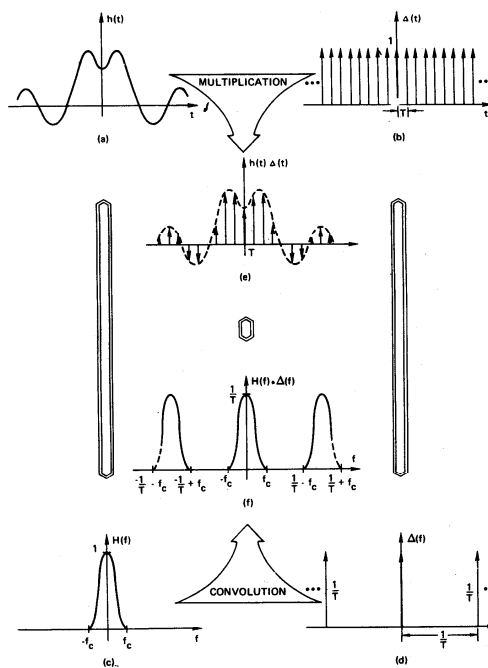


Figure 5-3. Graphical frequency convolution theorem development of the Fourier transform of a sampled waveform.

周元昉

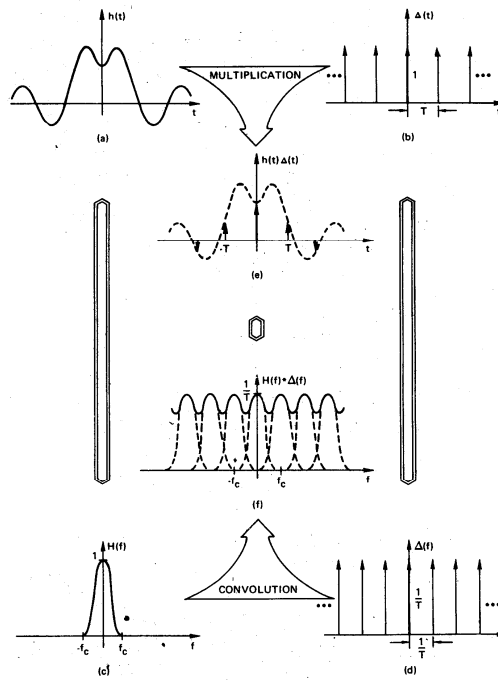


Figure 5-4. Aliased Fourier transform of a waveform sampled at an insufficient rate.

周元昉

Sampling theorem

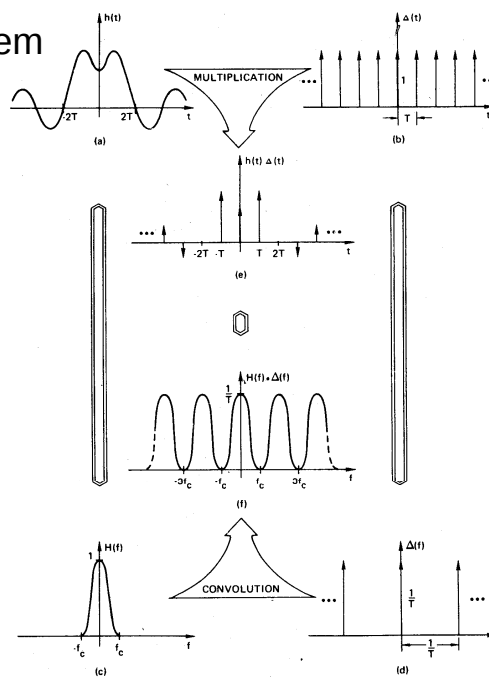


Figure 5-5. Fourier transform of a waveform sampled at the Nyquist sampling rate.

周元昉

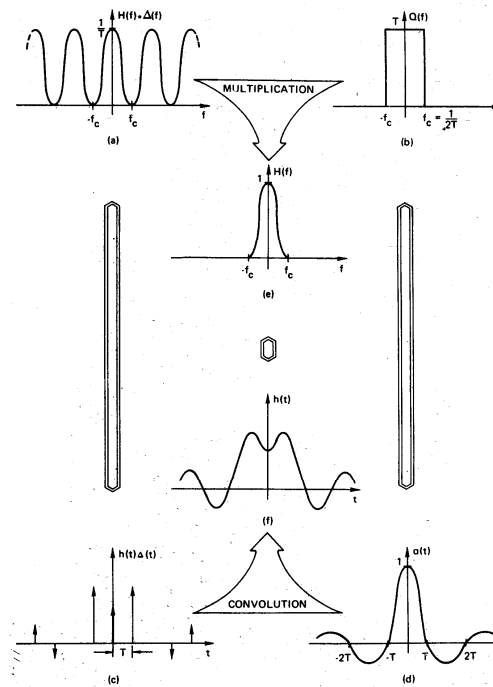


Figure 5-6. Graphical derivation of the sampling theorem.

周元昉

The discrete Fourier transform

$$h(t)\Delta_0(t) = h(t) \sum_{k=-\infty}^{\infty} \delta(t - kT) = \sum_{k=-\infty}^{\infty} h(kT)\delta(t - kT)$$

$$h(t)\Delta_0(t)x(t) = \sum_{k=0}^{N-1} h(kT)\delta(t - kT)$$

$$\Delta_1(t) = T_0 \sum_{r=-\infty}^{\infty} \delta(t - rT_0)$$

$$[h(t)\Delta_0(t)x(t)] * \Delta_1(t) = \hat{h}(t) = T_0 \sum_{r=-\infty}^{\infty} \sum_{k=0}^{N-1} h(kT)\delta(t - kT - rT_0)$$

周元昉

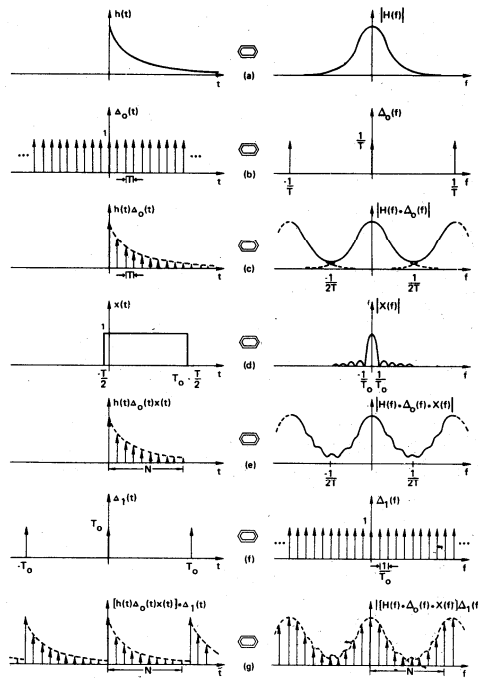


Figure 6-2. Graphical derivation of the discrete Fourier transform pair.

周元昉

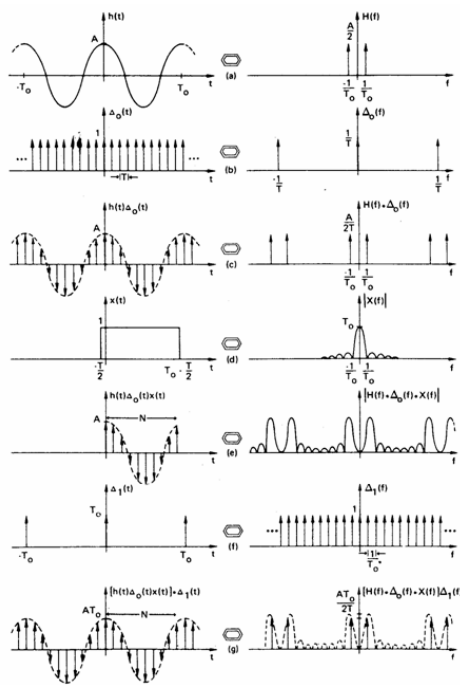


Figure 6-3. Discrete Fourier transform of a band-limited periodic waveform: truncation interval equal to period

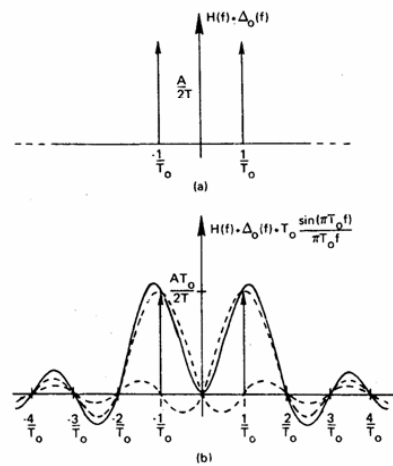


Figure 6-4. Expanded illustration of the convolution of Fig. 6-3(e).

周元昉

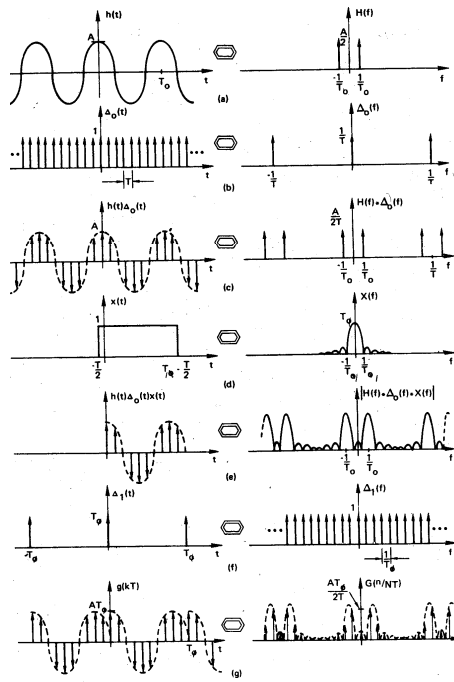


Figure 6-5. Discrete Fourier transform of a band-limited periodic waveform: truncation interval not equal to period.

周元昉

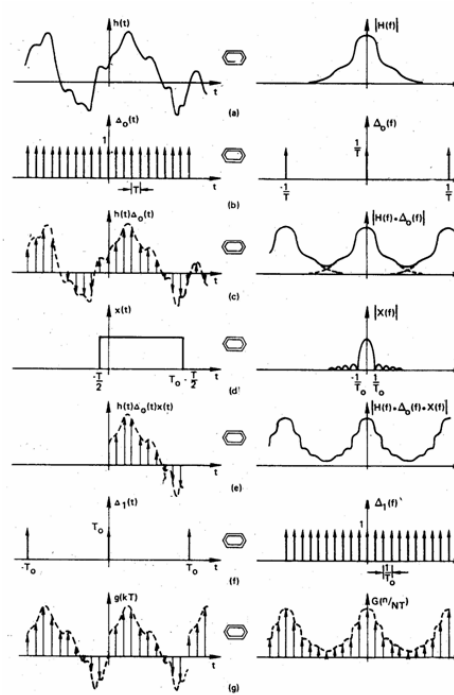
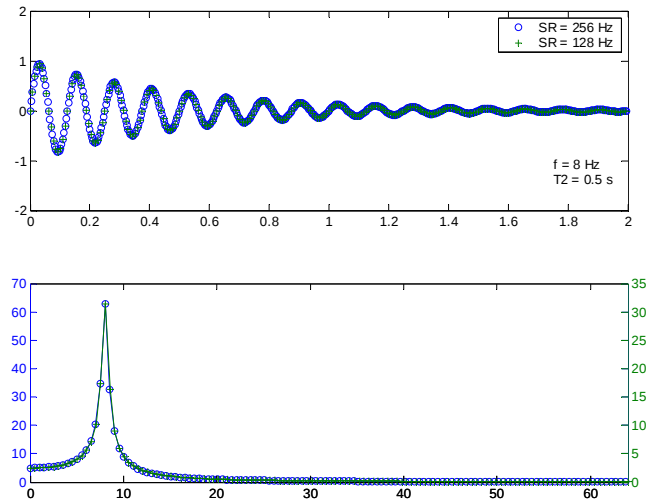


Figure 6-8. Discrete Fourier transform of a general waveform.

周元昉

Effect of changing sample rate

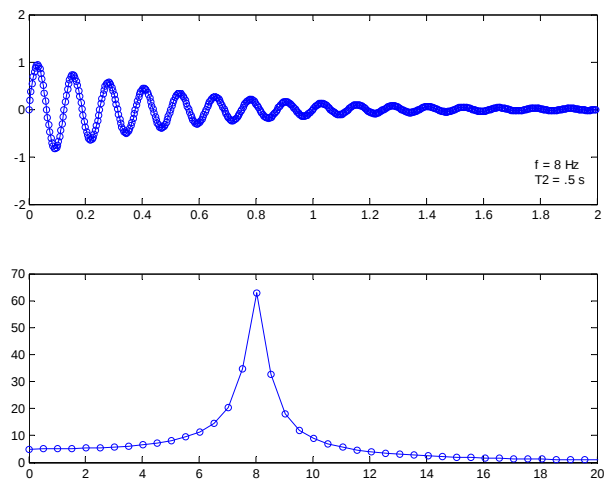


周元昉

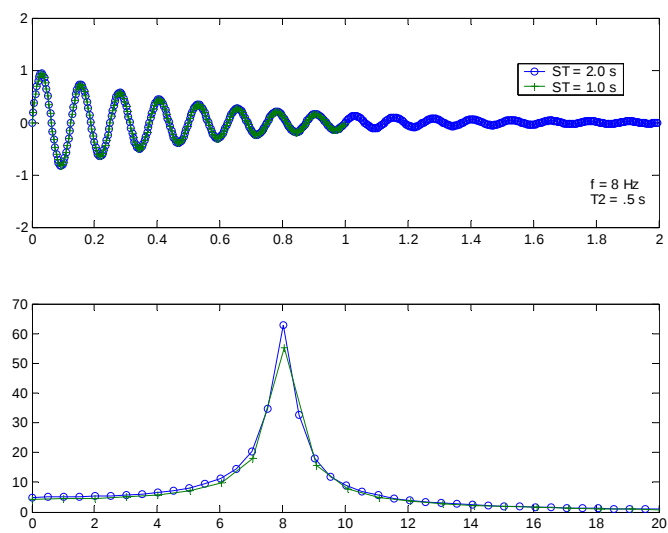
- Lowering the sample rate:
 - Reduces the Nyquist frequency, which
 - Reduces the maximum measurable frequency
 - Does not affect the frequency resolution

周元昉

Effect of changing sampling duration



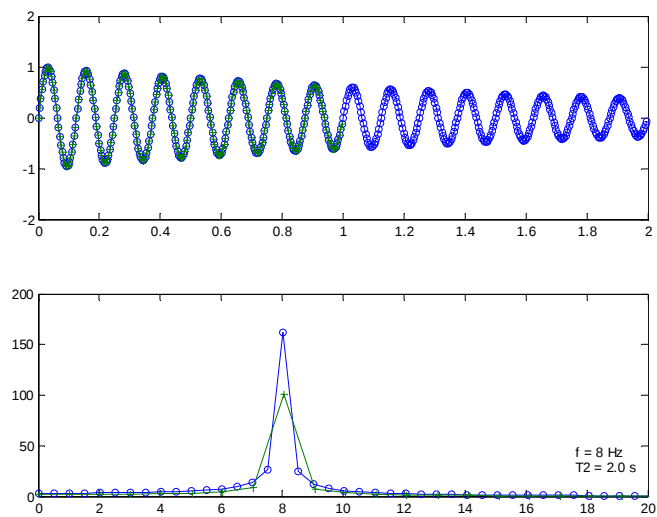
周元昉



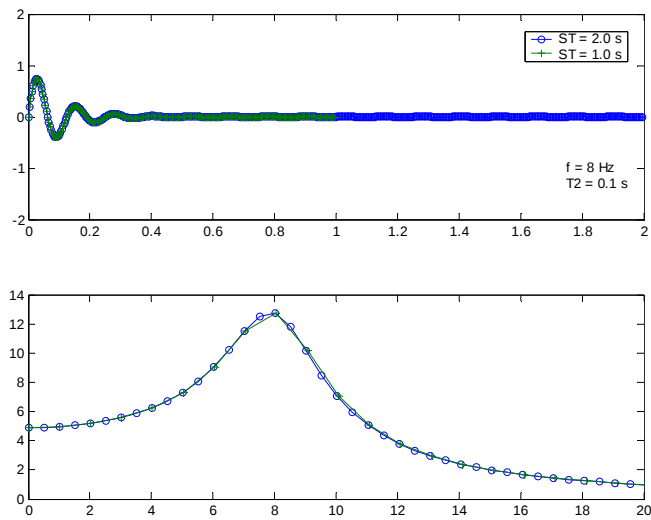
周元昉

- Reducing the sampling duration:
 - Lowers the frequency resolution
 - Does not affect the range of frequencies you can measure

周元昉

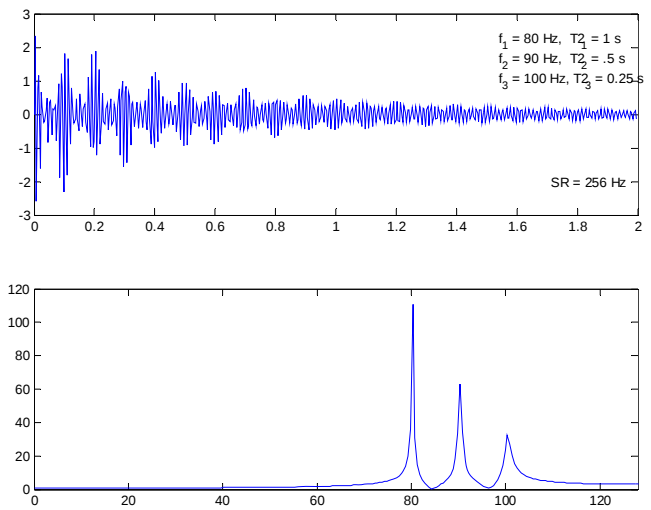


周元昉

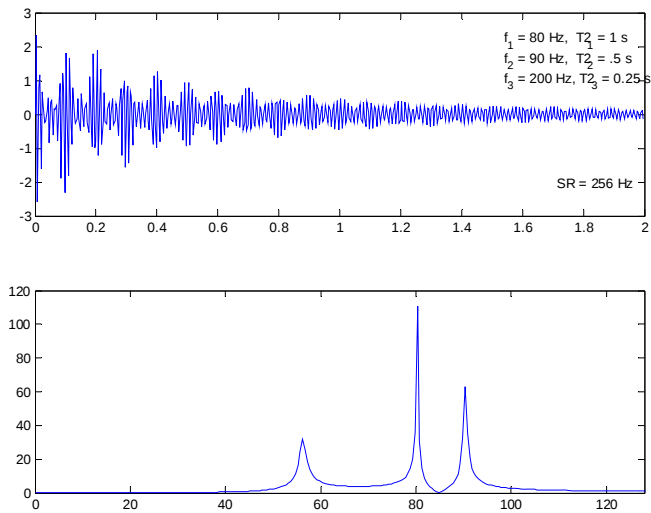


周元昉

Measuring multiple frequencies



周元昉



周元昉

The radix-2 decimation-in-time (DIT) FFT

- The Cooley-Tukey algorithm

$$X\left(\frac{n}{NT}\right) = \sum_{k=0}^{N-1} x_0(kT) e^{-j2\pi nk/N} \quad n = 0, 1, \dots, N-1$$

$$\frac{n}{NT} \rightarrow n; kT \rightarrow k; W = e^{-j2\pi/N}$$

$$X(n) = \sum_{k=0}^{N-1} x_0(k) W^{nk} \quad n = 0, 1, \dots, N-1$$

$$(1) N = 2^2 = 4$$

$$k = 0, 1, 2, 3 \rightarrow k = (k_1, k_0) = 00, 01, 10, 11$$

$$n = 0, 1, 2, 3 \rightarrow n = (n_1, n_0) = 00, 01, 10, 11$$

$$X(n_1, n_0) = \sum_{k_0=0}^1 \sum_{k_1=0}^1 x_0(k_1, k_0) W^{(2n_1+n_0)(2k_1+k_0)}$$

周元昉

$$\begin{aligned}
W^{(2n_1+n_0)(2k_1+k_0)} &= W^{(2n_1+n_0)2k_1} W^{(2n_1+n_0)k_0} \\
&= W^{[4n_1k_1]} W^{2n_0k_1} W^{(2n_1+n_0)k_0} \\
&= W^{2n_0k_1} W^{(2n_1+n_0)k_0} \\
W^{4n_1k_1} &= [W^4]^{n_1k_1} = [e^{-j2\pi 4/4}]^{n_1k_1} = 1^{n_1k_1} = 1
\end{aligned}$$

$$\Rightarrow X(n_1, n_0) = \sum_{k_0=0}^1 \left[\sum_{k_1=0}^1 x_0(k_1, k_0) W^{2n_0k_1} \right] W^{(2n_1+n_0)k_0}$$

$$x_1(n_0, k_0) = \sum_{k_1=0}^1 x_0(k_1, k_0) W^{2n_0k_1}$$

$$x_1(0,0) = x_0(0,0) + x_0(1,0)W^0$$

$$x_1(0,1) = x_0(0,1) + x_0(1,1)W^0$$

$$x_1(1,0) = x_0(0,0) + x_0(1,0)W^2$$

$$x_1(1,1) = x_0(0,1) + x_0(1,1)W^2$$

周元昉

$$\begin{bmatrix} x_1(0,0) \\ x_1(0,1) \\ x_1(1,0) \\ x_1(1,1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & W^0 & 0 \\ 0 & 1 & 0 & W^0 \\ 1 & 0 & W^2 & 0 \\ 0 & 1 & 0 & W^2 \end{bmatrix} \begin{bmatrix} x_0(0,0) \\ x_0(0,1) \\ x_0(1,0) \\ x_0(1,1) \end{bmatrix}$$

$$x_2(n_0, n_1) = \sum_{k_0=0}^1 x_1(n_0, k_0) W^{(2n_1+n_0)k_0}$$

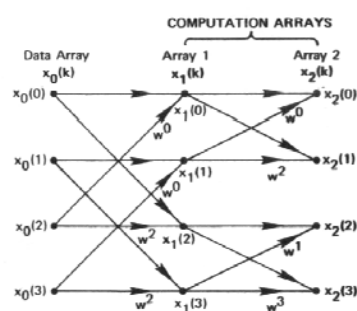
$$\begin{bmatrix} x_2(0,0) \\ x_2(0,1) \\ x_2(1,0) \\ x_2(1,1) \end{bmatrix} = \begin{bmatrix} 1 & W^0 & 0 & 0 \\ 1 & W^2 & 0 & 0 \\ 0 & 0 & 1 & W^1 \\ 0 & 0 & 1 & W^3 \end{bmatrix} \begin{bmatrix} x_1(0,0) \\ x_1(0,1) \\ x_1(1,0) \\ x_1(1,1) \end{bmatrix}$$

$$x_2(n_0, n_1) \Rightarrow X(n_1, n_0)$$

周元昉

$$X(n_1, n_0) = \sum_{k_0=0}^1 \left[\sum_{k_1=0}^1 x_0(k_1, k_0) W^{2n_0 k_1} \right] W^{(2n_1+n_0)k_0}$$

$$\begin{bmatrix} X(0,0) \\ X(1,0) \\ X(0,1) \\ X(1,1) \end{bmatrix} = \begin{bmatrix} 1 & W^0 & 0 & 0 \\ 1 & W^2 & 0 & 0 \\ 0 & 0 & 1 & W^1 \\ 0 & 0 & 1 & W^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & W^0 & 0 \\ 0 & 1 & 0 & W^0 \\ 1 & 0 & W^2 & 0 \\ 0 & 1 & 0 & W^2 \end{bmatrix} \begin{bmatrix} x_0(0,0) \\ x_0(0,1) \\ x_0(1,0) \\ x_0(1,1) \end{bmatrix}$$



周元昉

$$(2) N = 8 = 2^3$$

$$X(n) = \sum_{k=0}^{N-1} x_0(k) W^{nk}$$

$$\rightarrow X(n_2, n_1, n_0) = \sum_{k_0=0}^1 \sum_{k_1=0}^1 \sum_{k_2=0}^1 x_0(k_2, k_1, k_0) W^{(4n_2+2n_1+n_0)(4k_2+2k_1+k_0)}$$

$$W^{(4n_2+2n_1+n_0)(4k_2+2k_1+k_0)}$$

$$= W^{(4n_2+2n_1+n_0)4k_2} W^{(4n_2+2n_1+n_0)2k_1} W^{(4n_2+2n_1+n_0)k_0}$$

$$\Rightarrow W^{(4n_2+2n_1+n_0)4k_2} = [W^{8(n_1 k_2)}] [W^{8(n_2 k_2)}] W^{4n_0 k_2} = W^{4n_0 k_2}$$

$$\Rightarrow W^{(4n_2+2n_1+n_0)2k_1} = [W^{8(n_2 k_1)}] W^{(2n_1+n_0)2k_1} = W^{(2n_1+n_0)2k_1}$$

$$W^8 = [e^{-j2\pi/8}]^8 = 1$$

周元昉

$$\begin{aligned}
X(n_2, n_1, n_0) &= \sum_{k_0=0}^1 \sum_{k_1=0}^1 \sum_{k_2=0}^1 x_0(k_2, k_1, k_0) W^{(4n_2+2n_1+n_0)(4k_2+2k_1+k_0)} \\
&= \sum_{k_0=0}^1 \left[\sum_{k_1=0}^1 \left[\sum_{k_2=0}^1 x_0(k_2, k_1, k_0) W^{4n_0k_2} \right] W^{(2n_1+n_0)2k_1} \right] W^{(4n_2+2n_1+n_0)k_0}
\end{aligned}$$

$$x_1(n_0, k_1, k_0) = \sum_{k_2=0}^1 x_0(k_2, k_1, k_0) W^{4n_0k_2}$$

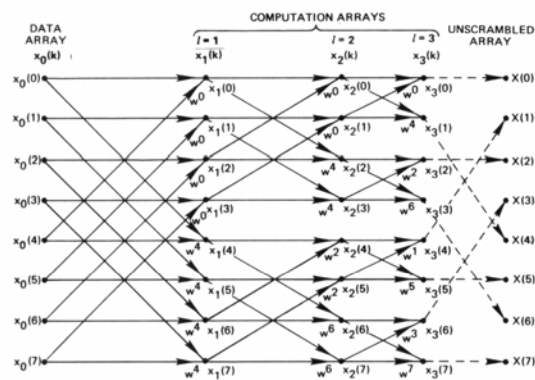
$$x_2(n_0, n_1, k_0) = \sum_{k_2=0}^1 x_1(n_0, k_1, k_0) W^{(2n_1+n_0)2k_1}$$

$$x_3(n_0, n_1, n_2) = \sum_{k_2=0}^1 x_2(n_0, n_1, k_0) W^{(4n_2+2n_1+n_0)k_0}$$

$$x_3(n_0, n_1, n_2) \Rightarrow X(n_2, n_1, n_0)$$

周元昉

N=8



周元昉

$$(3)N = 2^\gamma$$

$$X(n) = \sum_{k=0}^{N-1} x_0(k)W^{nk}$$

$$\Rightarrow X(n_{\gamma-1}, n_{\gamma-2}, \dots, n_0) = \sum_{k_0=0}^1 \sum_{k_1=0}^1 \dots \sum_{k_{\gamma-1}=0}^1 x_0(k_{\gamma-1}, k_{\gamma-2}, \dots, k_0)W^{nk}$$

$$n = 2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0$$

$$k = 2^{\gamma-1}k_{\gamma-1} + 2^{\gamma-2}k_{\gamma-2} + \dots + k_0$$

$$W^{nk} = W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)(2^{\gamma-1}k_{\gamma-1} + 2^{\gamma-2}k_{\gamma-2} + \dots + k_0)}$$

$$= W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)2^{\gamma-1}k_{\gamma-1}}$$

$$\times W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)2^{\gamma-2}k_{\gamma-2}}$$

$$\times \dots W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)k_0}$$

周元昉

$$W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)2^{\gamma-1}k_{\gamma-1}}$$

$$= [W^{2^\gamma(2^{\gamma-2}n_{\gamma-1}k_{\gamma-1})}] [W^{2^\gamma(2^{\gamma-3}n_{\gamma-2}k_{\gamma-1})}] \dots [W^{2^\gamma(n_1k_{\gamma-1})}] W^{2^{\gamma-1}(n_0k_{\gamma-1})}$$

$$= W^{2^{\gamma-1}(n_0k_{\gamma-1})}$$

$$W^{2^\gamma} = W^N = [e^{-j2\pi/N}]^N = 1$$

$$W^{(2^{\gamma-1}n_{\gamma-1} + 2^{\gamma-2}n_{\gamma-2} + \dots + n_0)2^{\gamma-2}k_{\gamma-2}}$$

$$= [W^{2^\gamma(2^{\gamma-3}n_{\gamma-1}k_{\gamma-2})}] [W^{2^\gamma(2^{\gamma-4}n_{\gamma-2}k_{\gamma-2})}] \dots W^{2^{\gamma-1}(n_1k_{\gamma-2})} W^{2^{\gamma-2}(n_0k_{\gamma-2})}$$

$$= W^{2^{\gamma-1}(n_1k_{\gamma-2})} W^{2^{\gamma-2}(n_0k_{\gamma-2})}$$

$$= W^{(2n_1 + n_0)2^{\gamma-2}k_{\gamma-2}}$$

周元昉

$$X(n_{\gamma-1}, n_{\gamma-2}, \dots, n_0) = \sum_{k_0=0}^1 \sum_{k_1=0}^1 \dots \sum_{k_{\gamma-1}=0}^1 x_0(k_{\gamma-1}, k_{\gamma-2}, \dots, k_0) \\ \times W^{2^{\gamma-1}(n_0 k_{\gamma-1})} W^{(2n_1 + n_0)2^{\gamma-2} k_{\gamma-2}} \dots W^{(2^{\gamma-1} n_{\gamma-1} + 2^{\gamma-2} n_{\gamma-2} + \dots + n_0) k_0}$$

$$x_1(n_0, k_{\gamma-2}, \dots, k_0) = \sum_{k_{\gamma-1}=0}^1 x_0(k_{\gamma-1}, k_{\gamma-2}, \dots, k_0) W^{2^{\gamma-1}(n_0 k_{\gamma-1})}$$

$$x_2(n_0, n_1, \dots, k_0) = \sum_{k_{\gamma-2}=0}^1 x_1(n_0, k_{\gamma-2}, \dots, k_0) W^{(2n_1 + n_0)2^{\gamma-2} k_{\gamma-2}}$$

$$\vdots$$

$$x_{\gamma}(n_0, n_1, \dots, n_{\gamma-1}) = \sum_{k_0=0}^1 x_{\gamma-1}(n_0, n_1, \dots, n_{\gamma-1}) W^{(2^{\gamma-1} n_{\gamma-1} + 2^{\gamma-2} n_{\gamma-2} + \dots + n_0) k_0}$$

$$x_{\gamma}(n_0, n_1, \dots, n_{\gamma-1}) \Rightarrow X(n_{\gamma-1}, n_{\gamma-2}, \dots, n_0)$$

周元昉

直接計算：

$$X(n) = \sum_{k=0}^{N-1} x_0(k) W^{nk} \quad n = 0, 1, \dots, N-1$$

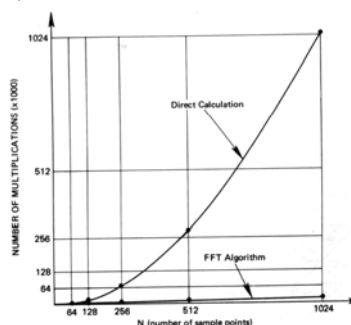
$\Rightarrow N^2$ 次複數乘法

基底2之FFT算法：

有 γ 個方程式，每方程式，有 N 個乘法

$\Rightarrow N \cdot \gamma$ 次複數乘法

$$\frac{N \cdot \gamma}{N^2} = \frac{\gamma}{N}$$



周元昉

$$\begin{aligned}
h(t) &= \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df \\
&= \int_{-\infty}^{\infty} [R(f) + jI(f)] [\cos(2\pi ft) + j \sin(2\pi ft)] df \\
&= \int_{-\infty}^{\infty} [R(f) \cos(2\pi ft) - I(f) \sin(2\pi ft)] df \\
&\quad + j \int_{-\infty}^{\infty} [R(f) \sin(2\pi ft) + I(f) \cos(2\pi ft)] df \\
&= \left[\int_{-\infty}^{\infty} [R(f) \cos(2\pi ft) - I(f) \sin(2\pi ft)] df \right. \\
&\quad \left. - j \int_{-\infty}^{\infty} [R(f) \sin(2\pi ft) + I(f) \cos(2\pi ft)] df \right]^* \\
&= \left[\int_{-\infty}^{\infty} R(f) e^{-j2\pi ft} df - j \int_{-\infty}^{\infty} I(f) e^{-j2\pi ft} df \right]^* \\
&= \left[\int_{-\infty}^{\infty} H^*(f) e^{-j2\pi ft} df \right]^*
\end{aligned}$$