# Chapter 5
# Mass and Energy Analysis of Control Volumes

# 控容系統質量及能量的分析

---

## 5-1 Conservation of Mass (質量守恆)

- *Mass , like energy, is a conserved property, and it cannot be created or destroyed.*
- *Mass and energy can be converted to each other .*

  *- mass is conserved even during chemical reactions*

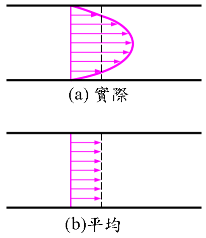
  *- E = $mc^2$ Einstein,*

  *through fusion and fission*

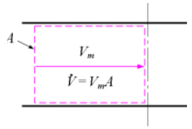
2 kg 氫 + 16 kg 氧 → 18 kg 水

---

## 質量守恆



流體在管路中**實際**和**平均**的速度外形(質量流率在兩種情形中皆相同)。



截面

$$\dot{V}(\mathrm{m^3/s}) = V_m(\mathrm{m/s}) \times A(\mathrm{m^2})$$

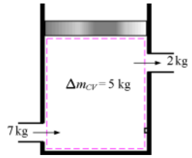
**體積流率**為單位時間內流經截面的流體體積。

$$\dot{m} = \rho\dot{V} = \rho V_m A$$

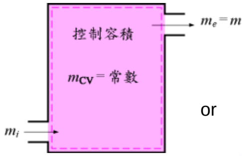
**質量流率**為單位時間內流經截面的流體質量。

---

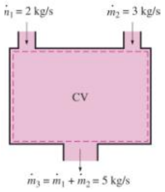
## 質量守恆



控制容積中之質量守恆原理。

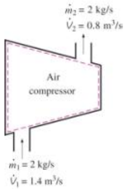


or



$$\Delta m_{cv} = 0 = m_{in} - m_{out}$$

$$\dot{m}_{cv} = 0 = \dot{m}_{in} - \dot{m}_{out}$$



在**穩流**過程中，進入控制容積的質量等於離開的質量。

**Note:** 穩流系統，體積流率不需守恆。

## 質量守恆

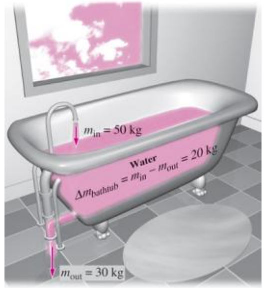


圖 **5-5** 一般浴缸中的質量守恆原理。

---

## The general mass balance principle for the lumped system:

*Change of mass in system* = *Sum of all inflow mass* - *Sum of all outflow mass* + *Net mass creation*

$$\Delta m_{system} = m_2 - m_1 = \sum_{i=1}^{N} m_{i,in} - \sum_{j=1}^{M} m_{j,out} \qquad + \Gamma_{creation}$$

$$m_{system} = \iiint_{CV} \rho dV, \quad \sum_{i=1}^{N} m_{i,in} - \sum_{j=1}^{M} m_{j,out} = -\int \left( \iint_{CS} \rho \vec{V} \cdot \vec{n} dA_{CS} \right) dt \quad \Gamma_{creation} = 0$$

*Rate of change of mass in system* = *Rate of all mass inflow* - *Rate of all mass outflow* + *Net rate of mass creation*

$$\left.\frac{dm}{dt}\right)_{system} = \sum_{i=1}^{N} \dot{m}_{i,in} - \sum_{j=1}^{M} \dot{m}_{j,out} \qquad + \dot{\Gamma}_{creation}$$

$$\frac{dm_{system}}{dt} = \frac{d}{dt} \iiint_{CV} \rho dV, \quad \sum_{i=1}^{N} \dot{m}_{i,in} - \sum_{j=1}^{M} \dot{m}_{j,out} = -\iint_{CS} \rho \vec{V} \cdot \vec{n} dA_{CS} \quad \dot{\Gamma}_{creation} = 0$$

---

## The general conservation of mass equation

$$\Delta m_{CV\,(system)} = \sum_{i=1}^{N} m_{i,in} - \sum_{j=1}^{M} m_{j,out}, \qquad \frac{dm_{CV}}{dt} = \sum_{i=1}^{N} \dot{m}_{i,in} - \sum_{j=1}^{M} \dot{m}_{j,out}$$

*Steady flow*

$$\sum_{i=1}^{N} m_{i,in} = \sum_{j=1}^{M} m_{j,out}, \qquad \sum_{i=1}^{N} \dot{m}_{i,in} = \sum_{j=1}^{M} \dot{m}_{j,out}$$

*+ single stream*

$$m_{in} = m_{out}, \text{ or } m_1 = m_2, \qquad \dot{m}_{in} = \dot{m}_{out}, \text{ or } \dot{m}_1 = \dot{m}_2$$

---

## 質量守恆

### Example 5.2

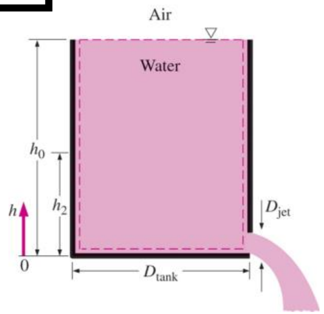


圖 **5-10** 範例5-2 示意圖。

# 5-2 Flow Work and the Energy of a Flowing Fluid (流動功與流動流體之能量)

- **Flow work:** *Unlike closed systems, control volumes involves mass flow across their boundaries, and some work is required to push the mass into or out of the control volume.*

  *- Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to PV.*

  $$W_{flow} = FL = PAL = PV \qquad (kJ)$$

- **Flow energy:** $e(u+k.e.+p.e.)+Pv$

## 流動功

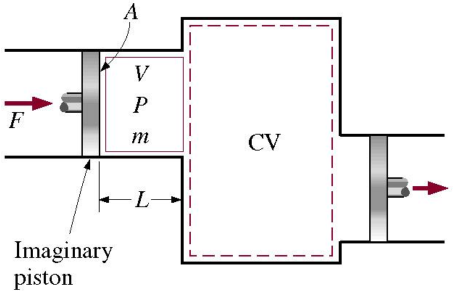
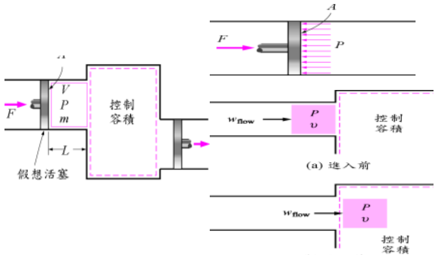


圖 **5-11** 流動功之示意圖。

## 流動功與流動流體之能量



流動功之示意圖。

$$F = PA$$

$$W_{flow} = FL = PAL = PV$$

$$\dot{W}_{flow} = P\dot{V},$$

$$\dot{m} = \rho\bar{V}A,$$

$$\dot{V} = \bar{V}A = \dot{m}/\rho = \dot{m}v$$

**流動功 – flow work**

$$w = \dot{W}/\dot{m} = P\dot{V}/\dot{m} = Pv$$

流動功等於 ***Pv***，為將**流體推入**或**推出**控制容積所需之能量。

## 流動功

***Flow work (energy) associated with unit flowing mass***

$$w = \dot{W}/\dot{m} = P\dot{V}/\dot{m} = Pv$$

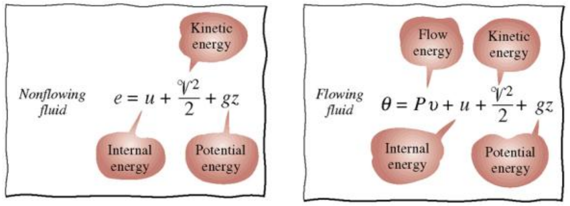
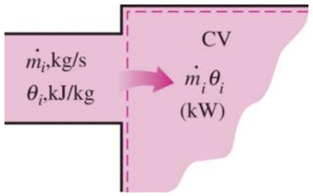


圖 **5-14** 靜止流體的總能量包含三個部分，流動流體有四個部分。

## Flow work and the energy of a flowing fluid

**The total energy of per unit mass of flowing fluid**



$$\theta = (u + ke + pe) + Pv$$

$$= h + \frac{V^2}{2} + gz$$

$$E_{mass} = m\theta$$

$$\dot{E}_{mass} = \dot{m}\theta$$

$$E_{mass} = \int \theta_i \delta m_i$$

圖 **5-15** 乘積 $\dot{m}_{in}\theta_{in}$為單位時間隨質量傳入控制容積的能量。

---

## 流功與流動流體之能量

**Example 5-3**

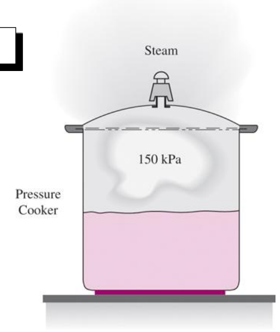
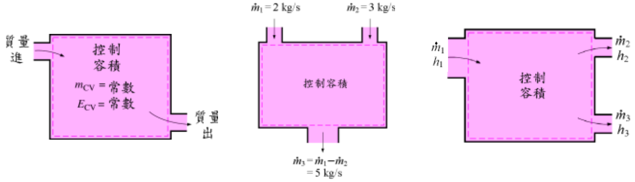


圖 **5-16** 範例 5-3 之示意圖。

---

## 5-3 Energy Analysis of Steady-Flow Systems (穩流系統之能量平衡)



穩流情況下，控制容積中的質量和能量，進口和出口的流體性質等均維持常數(不隨時間改變)。

---

## 穩流系統之能量平衡

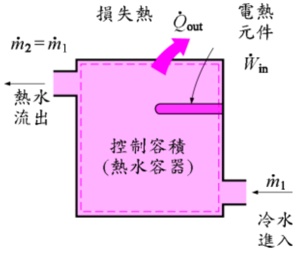


圖 **5-20** 穩定操作下之水的加熱器。

## 穩流系統之能量平衡

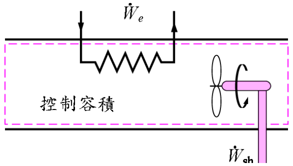
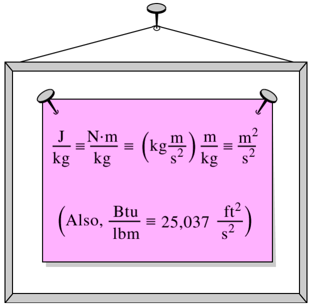


圖 **5-21** 穩定操作下，軸功和電功為一簡單可壓縮系統中功的其它形式。

## 穩流系統之能量平衡

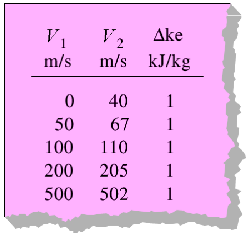


$$\frac{\mathrm{J}}{\mathrm{kg}} \equiv \frac{\mathrm{N \cdot m}}{\mathrm{kg}} \equiv \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\frac{\mathrm{m}}{\mathrm{kg}} \equiv \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

$$\left(\text{Also, } \frac{\mathrm{Btu}}{\mathrm{lbm}} \equiv 25{,}037\ \frac{\mathrm{ft}^2}{\mathrm{s}^2}\right)$$

圖 **5-22** 單位 **$m^2/s^2$ = J/kg** 。

## 穩流系統之能量平衡



| $V_1$ m/s | $V_2$ m/s | $\Delta$ke kJ/kg |
|---|---|---|
| 0 | 40 | 1 |
| 50 | 67 | 1 |
| 100 | 110 | 1 |
| 200 | 205 | 1 |
| 500 | 502 | 1 |

$$ke = \frac{V^2}{2}, \begin{cases} V = 50\,m/s, & 1.25 \\ 100, & 5 \\ 200, & 20 \\ 500, & 125 \end{cases} \quad kJ/kg$$

圖 **5-23** 在高速中，甚至微小速度之改變亦可能導致流體動能明顯的改變。

## The general energy balance principle for the lumped system:

Change E in C.V., $\Delta E$ = Heat transfer + Work transfer

+ Energy inflow due to fluid flow - Energy outflow due to fluid flow + Energy generation

$$\Delta E_{system} = Q - W + \sum_{i=1}^{N} m_{i,in}(u + ke + pe)_{in} - \sum_{j=1}^{M} m_{j,out}(u + ke + pe)_{out} + \Phi_{gen}$$

$$E_{system} = \iiint_{CV} \rho e dV, \qquad W = \text{all kinds of works} \qquad \Phi_{gen} = 0$$

$$\sum_{i=1}^{N} m_{i,in}(u + ke + pe)_{in} - \sum_{j=1}^{M} m_{j,out}(u + ke + pe)_{out} = -\int\left(\iint_{CS} \rho(u + ke + pe)\vec{V} \cdot \vec{n} dA_{CS}\right)dt$$

## The general energy balance principle for the lumped system: *in rate form*

*Rate of change E in C.V. dE/dt* = *Rate of heat transfer* + *Rate of work transfer*

+ *Energy inflow due to fluid flow* - *Energy outflow due to fluid flow* + *Rate of energy generation*

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_{i,in}(u + ke + pe)_{in} - \sum_{j=1}^{M} \dot{m}_{j,out}(u + ke + pe)_{out} + \dot{\Phi}_{gen}$$

$$\frac{dE_{CV}}{dt} = \frac{d}{dt}\iiint_{CV} \rho e dV, \quad \dot{W} = all\ kinds\ of\ rate\ of\ works, \quad \dot{\Phi}_{gen} = 0$$

$$\sum_{i=1}^{N} \dot{m}_{i,in}(u + ke + pe)_{in} - \sum_{j=1}^{M} \dot{m}_{j,out}(u + ke + pe)_{out} = -\iint_{CS} \rho(u + ke + pe)\vec{V} \cdot \vec{n} dA_{CS}$$

---

## *Special treatment on works:*

$$\dot{W} = all\ kinds\ of\ rate\ of\ works$$

$$\dot{W}_{total} = \dot{W}_{flowwork} + \dot{W}_{others}$$

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W}_{others} + \sum_{i=1}^{N} \dot{m}_{i,in}(u + Pv + ke + pe)_{in} - \sum_{j=1}^{M} \dot{m}_{j,out}(u + Pv + ke + pe)_{out}$$

*Or,*

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W}_{others} + \sum_{i=1}^{N} \dot{m}_{i,in}(h + ke + pe)_{in} - \sum_{j=1}^{M} \dot{m}_{j,out}(h + ke + pe)_{out}$$

*Or, in general , for simplification*

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_{i,in}(h + ke + pe)_{in} - \sum_{j=1}^{M} \dot{m}_{j,out}(h + ke + pe)_{out}$$

---

## *Recall: Energy balance for control volume system*

**Energy change of a system,** $\Delta E_{system} = E_{final} - E_{initial}$

where $E = U + KE + PE$

**Mechanisms of Energy Transfer,** $E_{in}$ and $E_{out}$

where $E_{in} = Q_{in} + W_{in} + E_{mass,in}$ $\quad E_{out} = Q_{out} + W_{out} + E_{mass,out}$

**Energy balance,** $\Delta E_{system} = E_{in} - E_{out}$

$$\Delta E_{system} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) + (E_{mass,in} - E_{mass,out})$$

$$E_{mass} = m(h + ke + pe), \quad h = u + Pv$$

**Energy balance for a** ***control volume*** **system,**

$$\Delta E_{system} = Q - W + \left[m_{in}(h + ke + pe)_{in} - m_{out}(h + ke + pe)_{out}\right]$$

$$\dot{E}_{system} = \dot{Q} - \dot{W} + \left[\dot{m}_{in}(h + ke + pe)_{in} - \dot{m}_{out}(h + ke + pe)_{out}\right]$$

---

## Summary *of the balance equations for control volume*

**Mass and energy balance** $\quad \Delta m_{cv} = m_{in} - m_{out}$

$$\Delta E_{system} = Q - W + \left[m_{in}(h + ke + pe)_{in} - m_{out}(h + ke + pe)_{out}\right]$$

**In rate form** $\quad \dot{m}_{cv} = \dot{m}_{in} - \dot{m}_{out}$

$$\dot{E}_{system} = \dot{Q} - \dot{W} + \left[\dot{m}_{in}(h + ke + pe)_{in} - \dot{m}_{out}(h + ke + pe)_{out}\right]$$

**穩流(steady flow)** $\quad \Delta m_{cv} = 0 = m_{in} - m_{out}$

$$\dot{m}_{cv} = 0 = \dot{m}_{in} - \dot{m}_{out}$$

$$\Delta E_{system} = 0 = Q - W + m\left[(h + ke + pe)_{in} - (h + ke + pe)_{out}\right]$$

$$\dot{E}_{system} = 0 = \dot{Q} - \dot{W} + \dot{m}\left[(h + ke + pe)_{in} - (h + ke + pe)_{out}\right]$$

*Note: the in-flow and out-flow works are separated from other kind of works*

# 5-4 若干穩流工程裝置

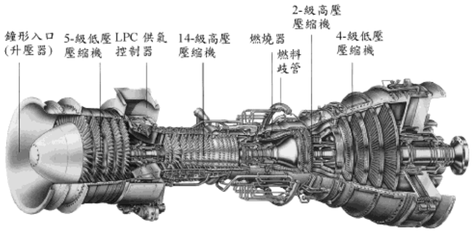


圖 **5-24** 用於發電的新式地上型氣渦輪機，此為奇異LM5000 渦輪機，長6.2 m、重12.5 噸，水蒸汽噴射，在3600 rpm 產生55.2 MW。

---

# 若干穩流工程裝置

## Nozzle (噴嘴) and Diffuser (升壓器)

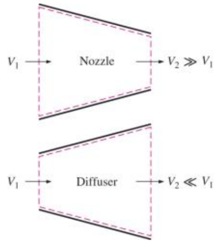


圖 **5-25** 噴嘴和升壓器(擴散器)之形狀設計成能使流體速度產生很大的改變，以致動能亦產生大的改變。

---

## Nozzle (噴嘴) and Diffuser (升壓器)

## Example 5.4 Diffuser

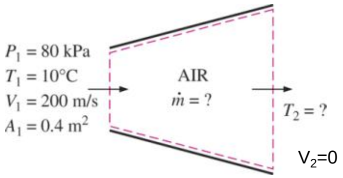


圖 **5-26** 範例 5-4 之示意圖。

---

## Nozzle (噴嘴) and Diffuser (升壓器)

## Example 5.5 Nozzle

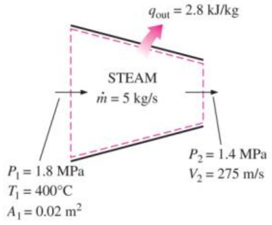


圖 **5-26A** 範例 5-5 之示意圖。

## 5-4 若干穩流工程裝置

### Turbine (渦輪機) and Compressor (壓縮機)

### Example 5.6  Compressor

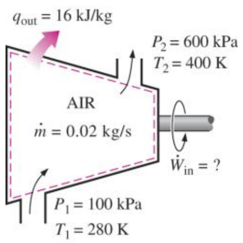


圖 **5-27** 範例 5-6 之示意圖。

---

### Turbine (渦輪機) and Compressor (壓縮機)

### Example 5.7  Turbine

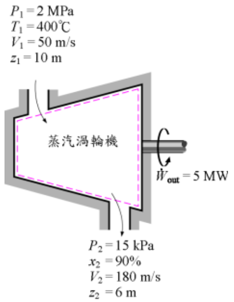


圖 **5-28** 範例 5-7 之示意圖。

---

## 5-4 若干穩流工程裝置

### Throttling Valves (節流閥)

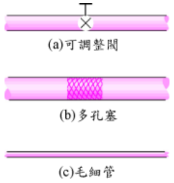

(a)可調整閥
(b)多孔塞
(c)毛細管

圖 **5-29** 節流閥為使流體產生大壓力降之裝置。

---

### Throttling Valves (節流閥)

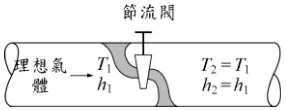


圖 **5-30** 節流過程中 ($h$ = 常數)，因為 $h$=$h(T)$ ，理想氣體之溫度不變。

## Throttling Valves (節流閥)

## Example 5.8  Expansion (throttling) Valve

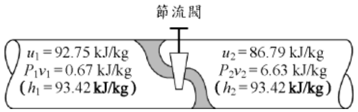
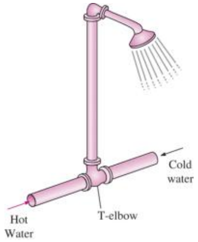


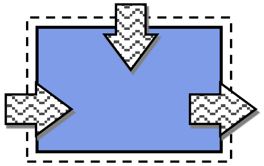
圖 **5-31** 在節流過程中，流體之焓(流動能+內能)維持常數，但是內能和流動能可互相轉換。

---

# 5-4 若干穩流工程裝置

## Mixing Chambers (混合室)





***Mixed fluids***

圖**5-32** 一般的T-肘蓮蓬頭可當作熱和冷流體的混合室。

---

## Mixing Chambers (混合室)

## Example 5.9  Mixing

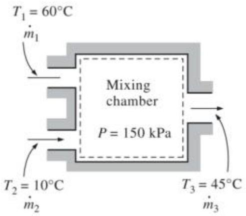


圖 **5-33** 範例 5-9 之示意圖。

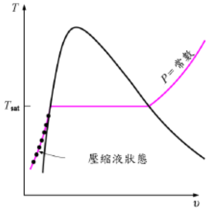


圖 **5-34** 一物質以低於已知壓力下的飽和溫度之壓縮液存在。

---

# 5-4 若干穩流工程裝置

## Heat Exchanger (熱交換器)

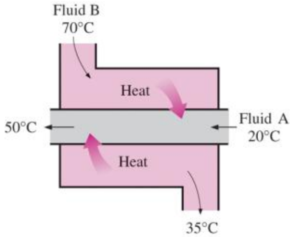
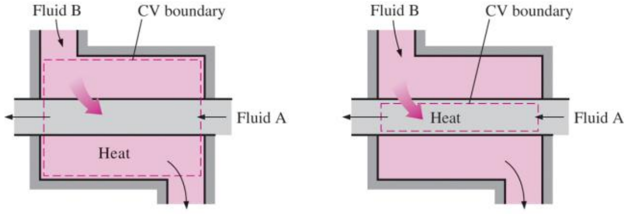


圖 **5-35** 一熱交換器可簡單如兩同心之管路。

## Heat Exchanger (熱交換器)



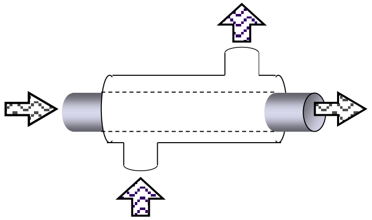
(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

(*b*) System: Fluid A ($Q_{CV} \neq 0$)

圖 **5-36** 熱交換器之熱交換可為零或非零，端賴如何選擇系統。

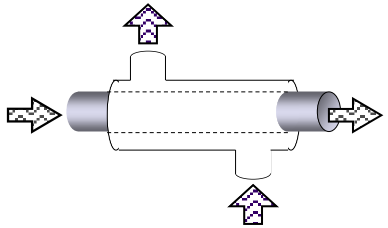
---

## Generic types of heat exchangers



***Two fluids, unmixed.***
***Shell and tube.***
***Parallel flow.***

***Two fluids, unmixed.***
***Shell and tube.***
***Counter flow.***



---

## Heat Exchanger (熱交換器)

## Example 5.10  Heat Exchanger

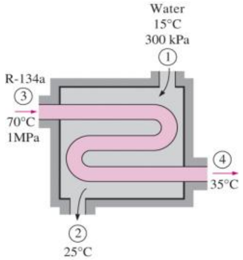


圖 **5-37** 範例 5-10 之示意圖。

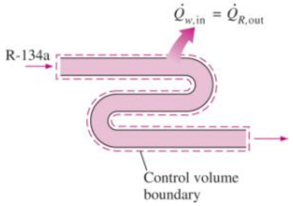


圖 **5-38** 熱交換器中，熱交換端賴所選擇之控制容積。

---

## 若干穩流工程裝置

### Pipe and Duct Flow

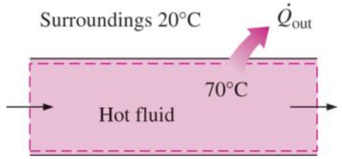


圖 **5-39** 由流經未絕熱的管路或管道中的熱流體傳至較冷環境的熱損失可能非常顯著。

## Pipe and Duct Flow

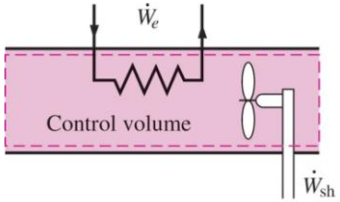


圖 **5-40** 管路或管道流可能在同時涉及超過一種形式的功。

---

## Pipe and Duct Flow

### Example 5.11 Electric Heating

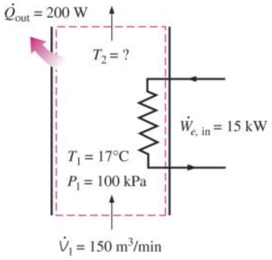


圖 **5-41** 範例 5-11 之示意圖。

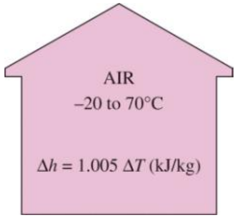


圖 **5-42** 對 −20 至 70℃ 的空氣，$\Delta h = C_P \Delta T$（$C_P = 1.005$ kJ/kg‧℃）的誤差小於 0.5%。

---

## 5-5 非穩流過程之能量平衡

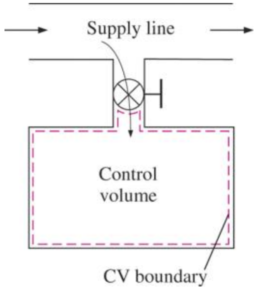


圖 **5-43** 由一供應管路裝載一剛性容器為一非穩流過程，因其涉及控制容積內之改變。

---

## 非穩流過程之能量平衡

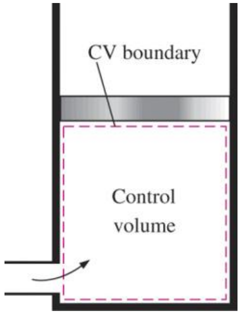


圖 **5-44** 在一非穩流過程中，控制容積之形狀和尺寸可能改變。

## 非穩流過程之能量平衡

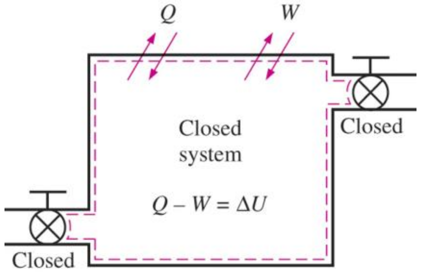


圖 **5-45** 當所有進口和出口被封閉時，一均匀流系統之能量方程式可簡化為一封閉系統。

## 非穩流過程之能量平衡

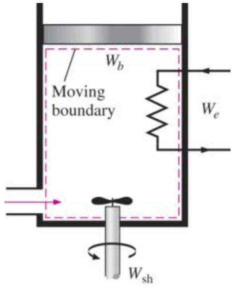
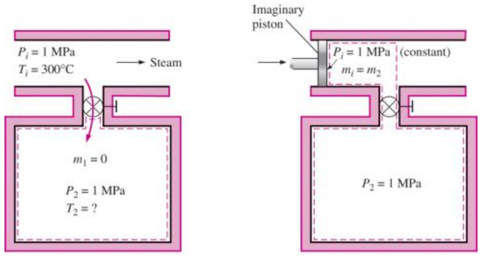


圖 **5-46** 一非穩流系統可同時涉及電、軸和邊界功。

## 非穩流過程之能量平衡

### Example 5.12  Charging Problem



(*a*) Flow of steam into an evacuated tank

(*b*) The closed-system equivalence

圖 **5-47** 範例 5-12 示意圖。

## 非穩流過程之能量平衡

### Example 5.12  Charging Problem

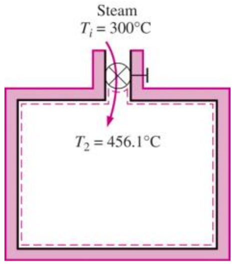


圖 **5-48** 由於流動能轉換成內能，當水蒸汽進入一容器，溫度由300℃升至456.2℃

## 非穩流過程之能量平衡

### Example 5.13  Cooking

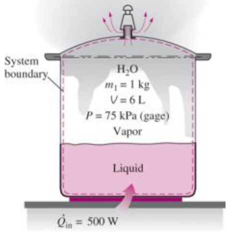
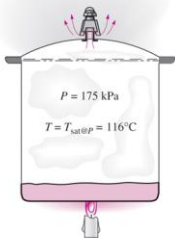
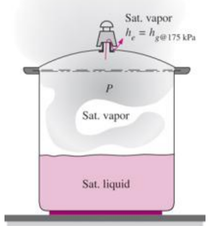


圖 **5-49** 範例 5-13 之示意圖。

## 非穩流過程之能量平衡



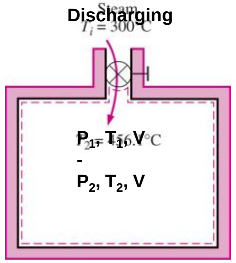
只要有水存在於壓力鍋中，飽和狀態將存在，而溫度將保持在飽和溫度。



在壓力鍋中，其中水蒸汽的焓為 *hg@P*(在一特定壓力下飽和汽之焓)。

## 非穩流過程之能量平衡

### Example of Discharging Problem



**Discharging**

**$P_1$, $T_1$, V**

**-**

**$P_2$, $T_2$, V**

**第五章習題**

**7, 12, 28, 35, 46,  52, 62, 72, 87, 96, 109, 114, 127, 139, 167**