A New Concept of Volumetric Error Analysis of Machine Tools Based on Abbé Principle

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Abstract

Abbé error is the inherent systematic error in all numerically controlled (NC) machine tools due to the fact that the displacement measuring axis is not in line with the cutting axis. Any angular error of the moving stage will result in the position offset from the commanded cutting point. The positioning error of any cutting point in space is called volumetric error. The angular error induced volumetric error, up to now however, is still considered with respect to the coordinate of the cutting point. In this paper, a new concept of deriving the volumetric error with respect to the Abbé offset of the cutting point is proposed. Experimental results verify the correctness of the proposed method.

Keywords: machine tool, volumetric error, Abbé error, error compensation

1 Introduction

Abbé error is the inherent systematic error in all numerically controlled (NC) machine tools. Abbé principle is regarded as the first principle in the design of precision positioning stages, machine tools, and measuring instruments [1]. It defines that the measuring apparatus is to be arranged in such a way that the distance to be measured is a straight-line extension of the graduation used as a scale. Bryan further made a generalized interpretation with that if the Abbé principle is not possible in the system design, either the slideway that transfer the displacement must be free of angular motion or the angular motion data must be obtained to compensate the Abbé error by software [2, 3].

Nowadays, most commercial machine tools and CMMs still cannot comply with Abbé principle because the scale axis is always parallel and offset to the moving axis. Any angular error of the moving stage will result in the position offset from the commanded cutting point. The positioning error of any cutting point in space is called volumetric error, which is composed of linear error terms, angular error induced 3D Abbé error terms and squareness error induced positional offset terms. Ever since Tlusty proposed the formulation of volumetric error of machine tool in 1980[4], all researchers and machine tool companies followed this equation to measure, formulate and compensate for the volumetric error of machine tools, for example, [5-7]. In this paper, a more physically sensible point of view is proposed to analyze the cause of volumetric error from Abbé principle. As a result, the Abbé error in the working volume can be fully compensated by experimental verification.

2 Volumetric error of machine tools

2.1 Existing method

The volumetric error of machine tool is normally caused by three sources, namely the linear errors of the moving stage in each axis, angular error induced deviation of cutting point in the working volume and the deviation of cutting point due to non-perpendicularity of any two axes. Tlustly first proposed the generalized equation as [4]

Volumetric error = Linear terms + Angular terms + Squareness terms (1)

In Eq. (1), the linear terms refer to the positioning error and two straightness errors of each moving stage, the angular terms indicate the grown linear errors caused by angular errors of each moving stage the squareness term is caused by non-perpendicular motion of any two axes. In this study, only the angular term is concerned. The general expression of this term is given by Eq. (2).

$$\begin{bmatrix} \delta_X(i) \\ \delta_Y(i) \\ \delta_Z(i) \end{bmatrix} = \begin{bmatrix} 0 & Z & -Y \\ Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_X(i) \\ \varepsilon_Y(i) \\ \varepsilon_Z(i) \end{bmatrix} \quad (2)$$

In this equation, $\delta_x(i)$ refers to the deviation of the cutting point from its nominal point in x direction during *i*-direction motion and i=x, y, z; $\varepsilon_x(i)$ denotes the angular error in x direction of *i*-direction motion; X, Y and Z are the coordinates of current cutting point, defined by offsets. For different structure configurations of machine tools, the effective offsets have to be found. It is seen that the deviation of the cutting point in space due to angular errors are proportionally enlarged by the coordinate where the cutter is locating. The geometrical relationship between the cutter point and its coordinate can be illustrated in **Fig. 1** in which the sensor point is not related. In other words, the Abbé offset is not considered in the volumetric error analysis. Physically, it is not correct.

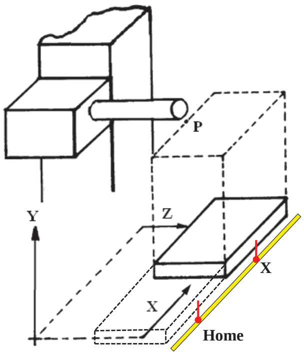

Fig. 1 Geometrical relationship of machine tool

2.2 Proposed method

According to Abbé principle, the position error of cutter location is calculated by the multiplication of the Abbé offset and the angular error, as shown in **Fig. 2** in 2D view. The guideway is non-straight and induces the pitch error (θ) of the stage that further generates the positioning error (δ) of the cutting point proportional to the amount of Abbé offset (L) from the sensor point. It results in $\delta = L \times \theta$, when θ is very small.

In the working volume, the cutting point error is the deviation in three directions, as shown in **Fig. 3**.

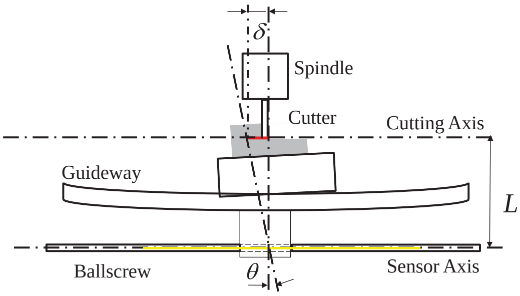

Fig. 2 Cutting point error due to Abbé error

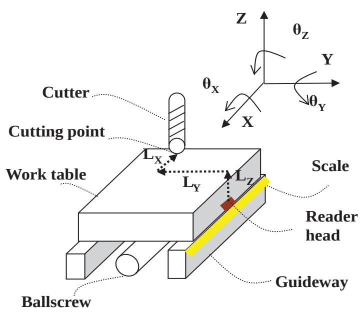

Fig. 3 Abbé error in 3D space

Therefore, the corresponding volumetric error can be expressed by

$$\begin{bmatrix} \delta_X \\ \delta_Y \\ \delta_Z \end{bmatrix} = \begin{bmatrix} -\theta_Z L_Y + \theta_Y L_Z \\ \theta_Z L_X - \theta_X L_Z \\ -\theta_Y L_X + \theta_X L_Y \end{bmatrix} \tag{3}$$

It is noted here that although Eq. (3) is similar to Eq. (2) in its form, however, the actual Abbé offset is adopted in use instead of the cutter coordinate adopted in Eq. (2). In practice, the Abbé offset has to be identified in each type of machine tool. The total volumetric errors of a three-axis machine tool caused by angular errors are the summation of components in each axis.

3 Example

Figure 4 shows the structure of a precision milling machine to be investigated. The Z- axis is mounted on the rail of the Column (X-axis). The work table is driven in Y-axis. All axes are equipped with liner scales as position feedback sensors. In this figure, the cutter is offset from the scale's read head, which is the sensor point in the Y-axis with three components: $L_{xY}(x)$, $L_{yY}(y)$ and $L_{zY}(z)$. These Abbé offsets are related to the instantaneous cutter position (x, y, z) and can be expressed by

$$\begin{aligned} L_{XY}(x) &= L_{XY0} - x \\ L_{yY}(y) &= L_{yY0} + y \\ L_{ZY}(z) &= L_{ZY0} - z \end{aligned} \tag{4}$$

In this equation, L_{xY0} is the distance between the cutter and reader head of Y axis when the cutter is moved at x=0. This offset will be reduced when the cutter moves in the x-axis. Similar expression can be applied to L_{yY0} and L_{zY0}. It can be seen, therefore, the actual Abbé offset is different from the coordinate of the cutter location, as expressed by Eq. (2).

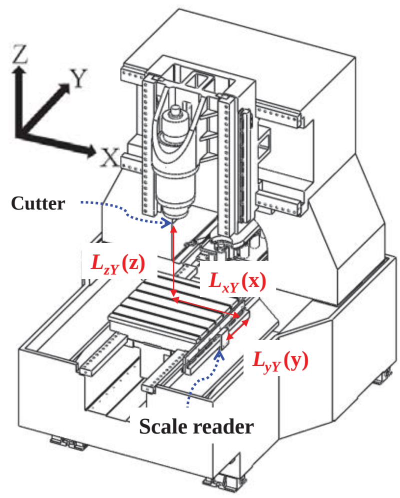

Fig. 4 Abbé offset of Y-axis motion

The corresponding volumetric errors caused by this Y-axis motion can be derived from Eq. (3) and Eq. (4) as

$$\begin{bmatrix} E_x(y) \\ E_y(y) \\ E_z(y) \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon_z(y) & \varepsilon_y(y) \\ \varepsilon_z(y) & 0 & -\varepsilon_x(y) \\ -\varepsilon_y(y) & \varepsilon_x(y) & 0 \end{bmatrix} \bullet \begin{bmatrix} L_{xY0} - x \\ L_{yY0} + y \\ L_{zY0} - z \end{bmatrix} \quad (5)$$

Figure 5 shows the Abbé offsets in x, y and z directions when the cutter moves along the X-axis. In this machine, the spindle moves with the X-axis motion. Hence, the relative distance between the cutter and the reader head of the X-axis scale is always constant during the X-motion. The offset in Y has the same nature while the offset in Z is dependent on the spindle's Z position. Therefore, the three Abbé offsets can be expressed by

$$\begin{aligned} L_{xX}(x) &= constant = L_{xX} \\ L_{yX}(x) &= constant = L_{yX} \\ L_{zX}(z) &= L_{zX0} - z \end{aligned} \quad (6)$$

The corresponding volumetric errors caused by this X-axis motion can be derived from Eq. (3) and Eq. (6) as

$$\begin{bmatrix} E_x(x) \\ E_y(x) \\ E_z(x) \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon_z(x) & \varepsilon_y(x) \\ \varepsilon_z(x) & 0 & -\varepsilon_x(x) \\ -\varepsilon_y(x) & \varepsilon_x(x) & 0 \end{bmatrix} \bullet \begin{bmatrix} L_{xX} \\ L_{yX} \\ L_{zX0} - z \end{bmatrix} \quad (7)$$

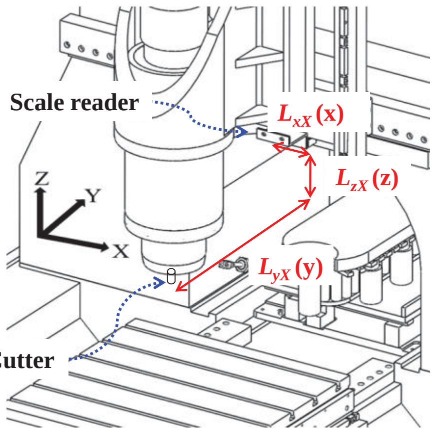

Fig. 5 Abbé offset of X-axis motion

Figure 6 shows the Abbé offsets in x, y and z directions when the cutter moves along the Z-axis. In this machine, the offset of the cutter to the reader head of the scale in Z-axis is always constant during the Z-motion.. Therefore, the three Abbé offsets can be expressed by

$$\begin{aligned} L_{xZ}(x) &= constant = L_{xZ} \\ L_{yZ}(y) &= constant = L_{yZ} \\ L_{zZ}(z) &= constant = L_{zZ} \end{aligned} \quad (8)$$

The corresponding volumetric errors caused by this X-axis motion can be derived from Eq. (3) and Eq. (8).

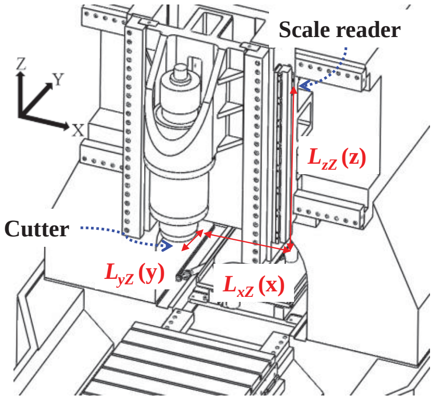

Fig. 6 Abbé offset of Z-axis motion

$$\begin{bmatrix} E_x(z) \\ E_y(z) \\ E_z(z) \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon_z(z) & \varepsilon_y(z) \\ \varepsilon_z(z) & 0 & -\varepsilon_x(z) \\ -\varepsilon_y(z) & \varepsilon_x(z) & 0 \end{bmatrix} \bullet \begin{bmatrix} L_{xZ} \\ L_{yZ} \\ L_{zZ} \end{bmatrix} \quad (9)$$

The total volumetric errors due to angular error terms of this three-axis machine tool can, therefore, be derived by the sum of Eqs. (5), (7) and (9). The component in each axis is

$$\begin{aligned} E(x) &= E_x(x) + E_x(y) + E_x(z) \\ E(y) &= E_y(x) + E_y(y) + E_y(z) \\ E(z) &= E_z(x) + E_z(y) + E_z(z) \end{aligned} \quad (10)$$

It has to be noted here that the Abbé offsets are dependent on the structure configuration of the machine tool and the constant terms of Abbé offsets have to be measured directly on the investigated machine. The highlight of this section points out that, to derive the volumetric errors due to angular errors the offset should be considered from the current sensor point, rather than the conventional machine origin point.

4 Experimental verification

From the above derived volumetric errors due to angular terms, it can be seen that the corresponding Abbé offsets during each axis motion can be realized from the machine installation. The angular terms, however, have to be measured, preferably embedded in each axis of the machine tool. **Figure 7** shows the schematic diagram of this concept in which a sensor module for the real-time detection of pitch, yaw and roll angles are mounted onto the X-axis of the linear stage. This angle sensor module has been developed by the author's group using low cost laser diode, optics and four-quadrant photo detectors [8]. It is not repeated here.

A test trial has been carried out on a small NC machine tool, which has built-in angular sensor module in each axis and has the PC-based controller with Abbé error compensator.

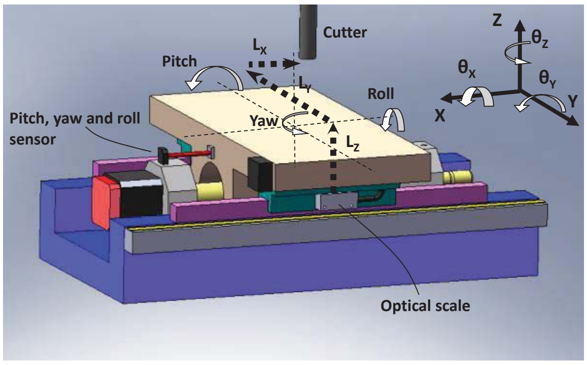

Fig. 7 Machine tool with embedded sensor module

The experimental setup is shown in **Fig. 8** for the X-axis motion. A laser interferometer of HP5529 was amounted at different Z heights of the spindle head as a calibration reference. Same procedure can also be conducted for the Y-motion. **Figure 9** and **Fig. 10** show the comparison of positioning errors with and without the Abbé error compensation in X- and Y-axis respectively. The carrier can be regarded as a rigid body motion. It is clearly seen that the positioning errors can be significantly reduced when the Abbé error compensation scheme is activated at any position.

A diagonal path test is then carried out to show simultaneous motion of X and Y axes. The experimental setup is shown in **Fig. 11**. Calibrated results, as shown in Fig. 12 and Fig. 13, prove that the Abbé error can be almost compensated.

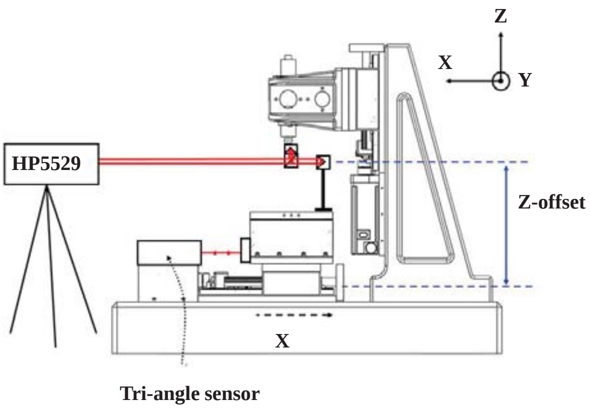

Fig. 8 Experimental setup for X-positioning test

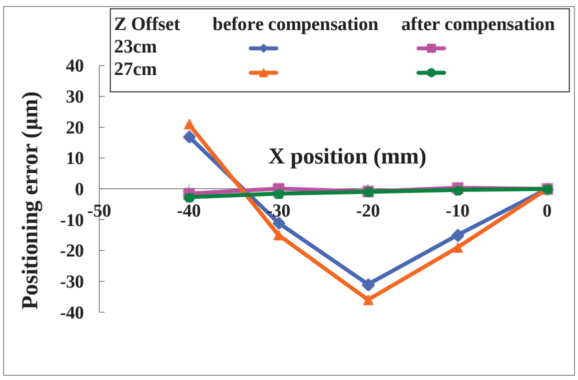

Fig. 9 Experimental results of X-positioning error calibration

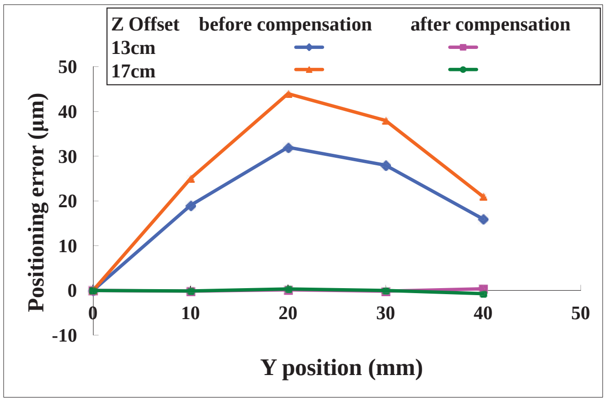

Fig. 10 Results of Y-positioning error calibration

5 Conclusions

In this paper, a new concept of volumetric error analysis of machine tools caused by angular errors are proposed. Different from the conventional method that regards the offset as the distance of the cutter location from the machine origin, the proposed method selects the offset as the distance from the cutter location to the sensor location of each axis. It is based on the well known Abbé principle and is more physically sensible. An example of the machine tool, which is under development, is taken for use to derive its volumetric errors due to angular terms. Experiments on a small three-axis machine tool have been conducted and the correctness of the proposed method has been verified by error compensation.

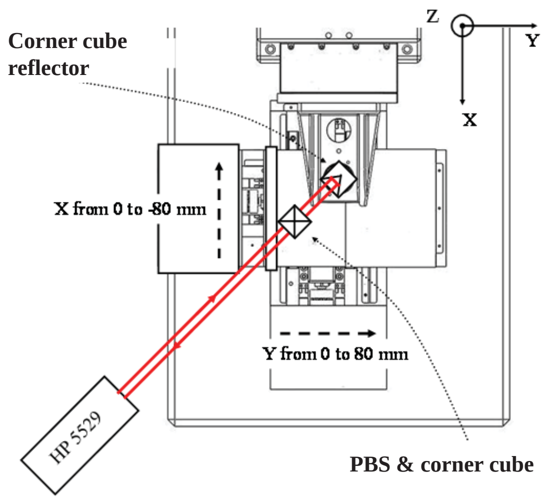

Fig. 11 Experimental setup for XY-positioning test

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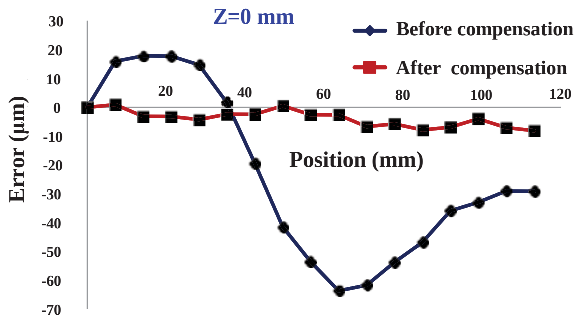

Fig. 12 Experimental results of XY-positioning error calibration at height Z=0 mm

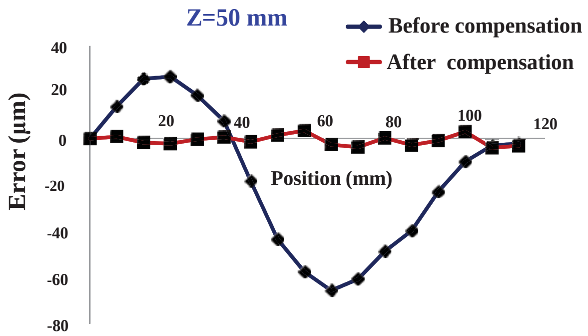

Fig. 13 Experimental results of XY-positioning error calibration at height Z=50 mm

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