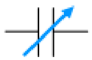
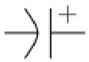
20140327 訊號處理

Outline

- 基本電學複習
- 濾波器
- 二極體電路
- 運算放大器
- 一些實例

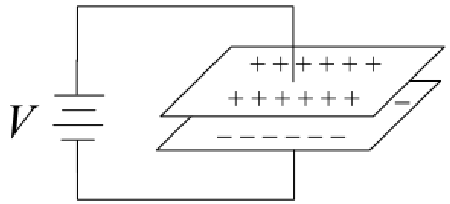
基本電學複習

Capacitor and Capacitance

- When a voltage is applied to two metallic plates very close to each other, but without contact, charges will be stored in the plates due to attraction of different polarity of charges. The device is called a capacitor with mark  or  (極性電容)
- The charge Q stored in a capacitor is given as

$$Q = CV \qquad (3)$$

  where C is the capacitance with unit Farad (1 Farad = 1 Coulomb of charge).
- Most circuits use capacitors in the order of μF to pF (pico F, i,e., 10^{-12}F)



$$V = Ed = \sigma d/\varepsilon$$

$$Q = \sigma A = C\sigma d/\varepsilon$$

$$C = \varepsilon A/d$$

ε: dielectric constant A: area,
d: distance between two plates

Voltage Change in a Capacitor

- To investigate voltage variation in a capacitor when charged with AC source, we can differentiate Eq. (3)

$$I = \frac{dQ}{dt} = C\frac{dV}{dt} \qquad (4)$$

- The AC voltage can be expressed as

$$V = V_0 \cos \omega t$$

where ω is the frequency (rad/s). It is convenient to rewrite the sinusoidal functions in terms of complex variable as

$$V = V_0 e^{j\omega t}$$

where $j = \sqrt{-1}$, and only the real-part make sense (discard the imaginary part). FYR: Euler formula: $e^{jx} = \cos x + j\sin x$. Plug into the above equation to obtain,

Characteristics of Capacitor

- We can figure out characteristics of a capacitor with AC operating as
  - There is a 90˚ phase difference (lagging) between the current and the applied AC voltage.
  - A capacitor can be regarded as a "*frequency-dependent resistor*" such that

$$\begin{cases} \mathcal{Z}_C \to \infty \text{ as } \omega \to 0 \\ \mathcal{Z}_C \to 0 \text{ as } \omega \to \infty \end{cases}$$

That is, the capacitor becomes an insulator for extremely low frequency, and an conductor for very high frequency.

For example, a BNC cable has an inherent capacitance of 28 pf/ft. If one measures 1.5V at 100 MHz using 1 meter BNC cable, the BNC cable will conduct a current approximate to $I = 0.1$ A. This quantity is astonishingly larger than imagination! Work with high-frequency is always costly and troublesomely.

Inductance and Inductor

- An inductor is a core of insulated metallic wire. A voltage will be produced in proportional to the rate change of current through it as

$$V = L\frac{dI}{dt}$$

where L is the inductance with unit Henry, H.   Similar to the process of using complex variables for the capacitor, we derive:

$$\mathcal{Z}_L = j\omega L, \quad V_L(t) = V_0 \cos\omega t = R(V_0 e^{j\omega t}) \qquad (6)$$

$$I(t) = \frac{V_0}{\omega L}\sin\omega t = -\frac{1}{L}R(\frac{V_0}{j\omega}e^{j\omega t}) = \frac{V_0}{\omega L}\cos(\omega t - \frac{\pi}{2})$$

- The equation reveals that the inductor is also a frequency-dependent element in opposite to the capacitor as

$$\begin{cases} \mathcal{Z}_L \to 0 \ \text{as} \ \ \omega \to 0 \\ \mathcal{Z}_L \to \infty \ \text{as} \ \ \omega \to \infty \end{cases}$$

濾波器

Voltage Divider

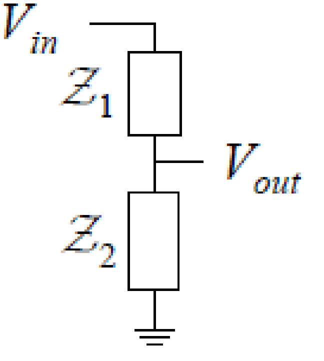
Very useful!

- As two resistors arranged in series as shown, the voltage output is simply

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}$$

- Such a device is called a *voltage divider* which can provide a friction of a source voltage.
- The voltage can be generalized to include reactant elements as

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$



Gain and Decibel

- Gain is always referred as the ratio of the output to the input of a device. For example, we can say the ratio of the output amplitude to the input amplitude as the gain of an oscillation system.
- For quantities span over several order of magnitude, it is convenient to express the gain in terms of decibel (denoted as dB):

$$\text{Gain(dB)} = 10\log_{10}\frac{\text{power output}}{\text{power input}}$$

- If the signal is measured in volt, the power is proportional to the square of a voltage, then (taking magnitude for complex variable)

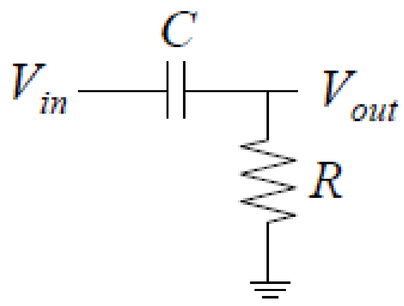
$$\text{Gain(dB)} = 20\log_{10}\frac{\text{voltage output}}{\text{voltage input}}$$

- Hence, –20 dB means that V_{out} decays to 10% of V_{in}.

High Pass Filter

- Consider a RC circuit wired as shown. The ratio of the output to the input voltages can be easily found using complex impedance

$$\frac{V_{out}}{V_{in}} = \frac{R}{R+Z_C} = \frac{R}{R+1/j\omega C} = \frac{j\omega RC}{1+j\omega RC}$$



- Examine some pertinent points:

$$\text{As} \quad \omega RC >> 1, \quad \frac{V_{out}}{V_{in}} \approx 1 \quad \rightarrow \quad \text{G(dB)} = 0$$

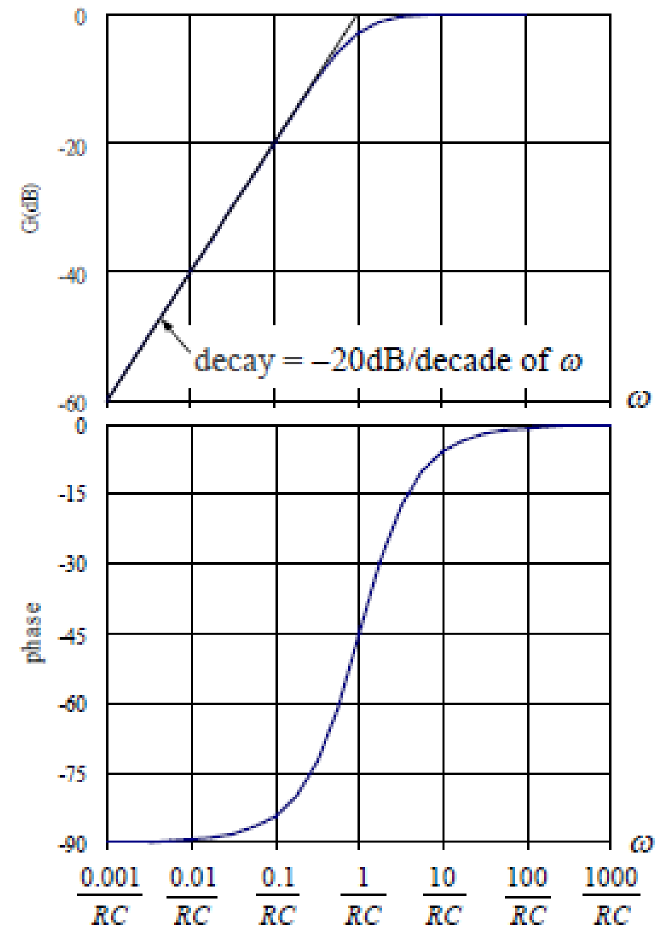
$$\text{As} \quad \omega RC = 1, \quad \frac{V_{out}}{V_{in}} = \frac{j}{1+j} \quad \rightarrow \quad \text{G(dB)} = -3.01$$

$$\text{As} \quad \omega RC << 1, \quad \frac{V_{out}}{V_{in}} \approx j\omega RC \quad \rightarrow \quad \text{G(dB)} = 20\log(\omega RC)$$

The circuit is referred as a high pass filter.

Bodé Diagram

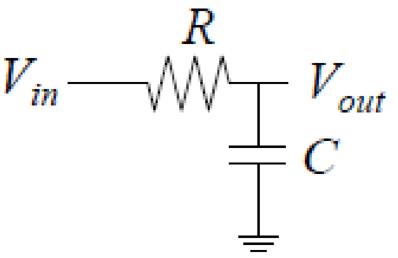
- Some essential facts:
  - G(dB) = 0 dictates no decay.
  - $\omega = 1/RC$ is called the *corner frequency*, or –3dB "breakpoint".
  - As $\omega \ll 1/RC$, a decade decrease in ω will result in –20dB in gain.
- We can plot the gain (dB) versus input frequency (ω) called the *Bodé diagram*.
- In addition to the ration of magnitudes, the *frequency response* also include a *phase difference* as shown.
- This circuit for high pass filter also acts as *a differentiator* for small values of *R* and *C*.



Low Pass Filter

- Consider a RC circuit wired in the other way as shown. The ratio of the output to the input voltages can be written as

$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{R+Z_C} = \frac{1/j\omega C}{R+1/j\omega C} = \frac{1}{1+j\omega RC}$$



- Let us check several pertinent points:

$$\text{As} \quad \omega RC << 1, \ \frac{V_{out}}{V_{in}} \approx 1 \rightarrow \text{G(dB)} = 0$$

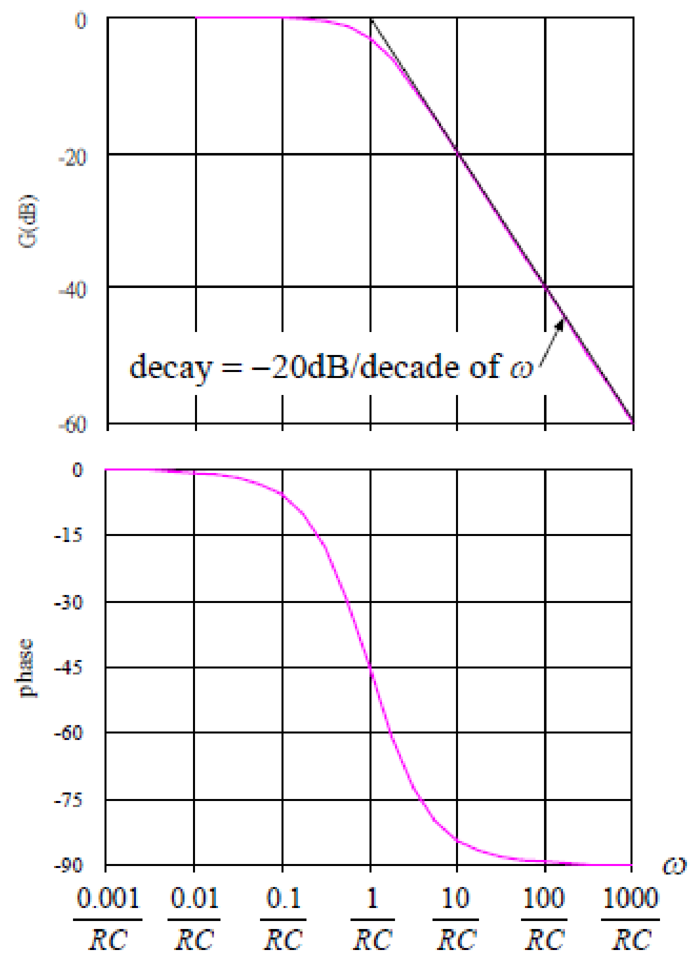
$$\text{As} \quad \omega RC = 1, \ \frac{V_{out}}{V_{in}} = \frac{1}{1+j} \rightarrow \text{G(dB)} = -3.01$$

$$\text{As} \quad \omega RC >> 1, \ \frac{V_{out}}{V_{in}} \approx j\omega RC \rightarrow \text{G(dB)} = 20\log(\omega RC)$$

- The circuit is called as a low pass filter.

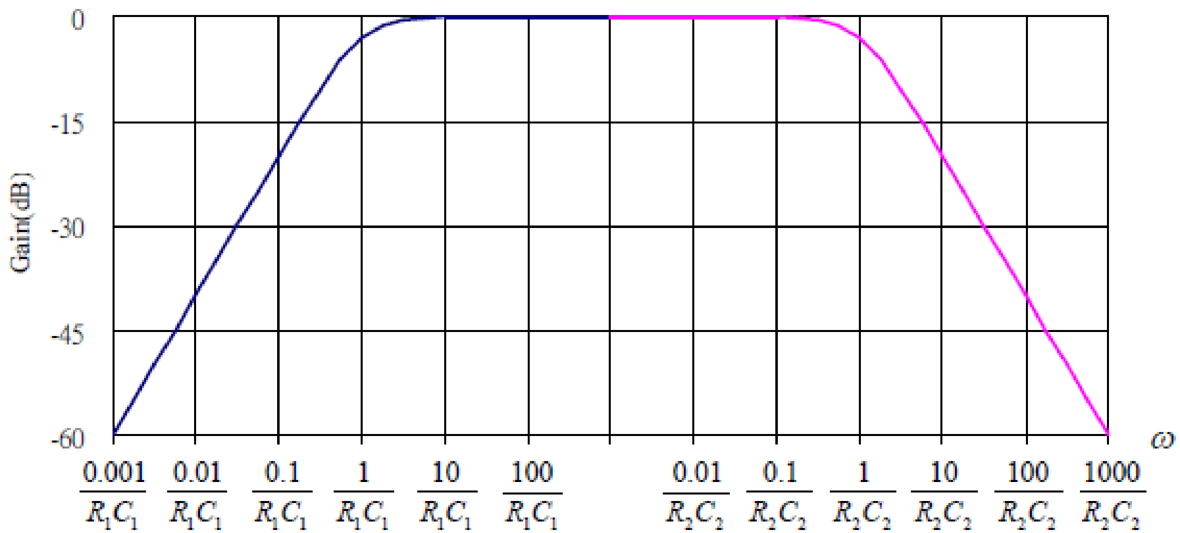
Low Pass Filter

- Some features:
  - G(dB) = 0 dictates no decay.
  - $\omega = 1/RC$ is also called the *corner frequency*, or –3dB "breakpoint".
  - As $\omega >> 1/RC$, a decade increase in ω will result in –20dB in gain.
- The *Bodé* diagram at right reveals *frequency response* of a low pass filter.
- The circuit for low pass filter can also serve as *an integrator* for large *RC* values.



Band Pass Filter

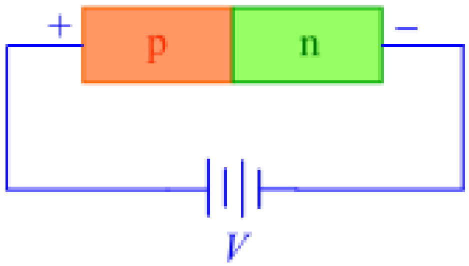
- If we choose two distinct corner frequencies $\omega_1 = 1/R_1C_1$ for the high pass filter and $\omega_2 = 1/R_2C_2$ for the high pass filter properly, we can design a band pass filter as shown below. Only frequencies in the plateau range will survive.
- All these filters are passive, owing to fact of no amplification.

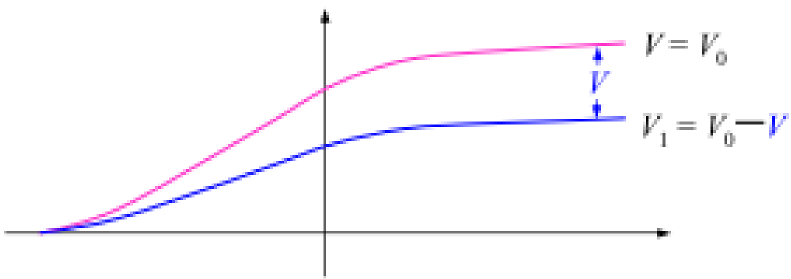


二極體電路

Forward Bias

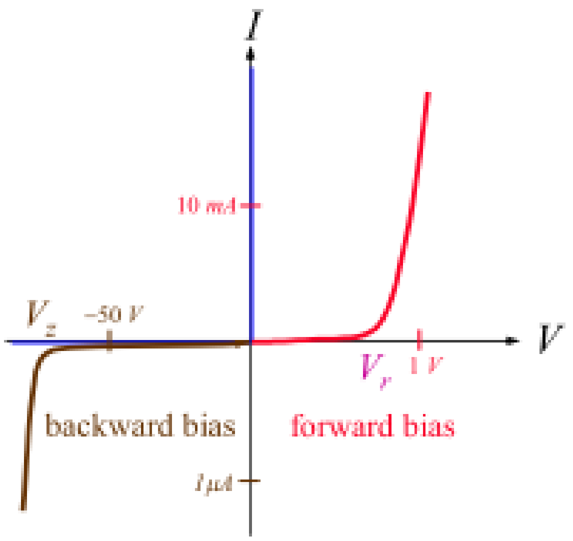
- As two types of semiconductors in simple contact, there should have no net current through the junction. Thus, $I_h^{gen} = I_h^{rec}$.
- This equilibrium will be changed if a voltage difference V is applied to the diode.
  - The applied positive voltage lowers down the "height" of electric potential, facilitating charges moving uphill. Such a voltage is called *forward bias*.
  - There is a large amount of holes in the p-type terminal. Hence, the applied positive voltage repels positive charges toward the n-type terminal.





Barrier and Breakdown Voltages

- An *ideal* diode passes unlimited current as a *one-way* switch if forward biased.
- In a *real* diode, the current is very low until the forward bias exceed a *barrier* (or *cut-in*, *turn-on*) *voltage*, V_r, beyond which the current grows nearly *exponentially* with the bias. The barrier voltage is about 0.6 to 0.7 V for Si, and 0.2 to 0.3 for Ge.



- Current of a diode under *reverse* bias is extremely small, in the order of 1 μA. As the voltage lowers down to the so-called breakdown voltage, V_z, the magnitude of current increase sharply. Such a property is exploited as *Zener* diodes.

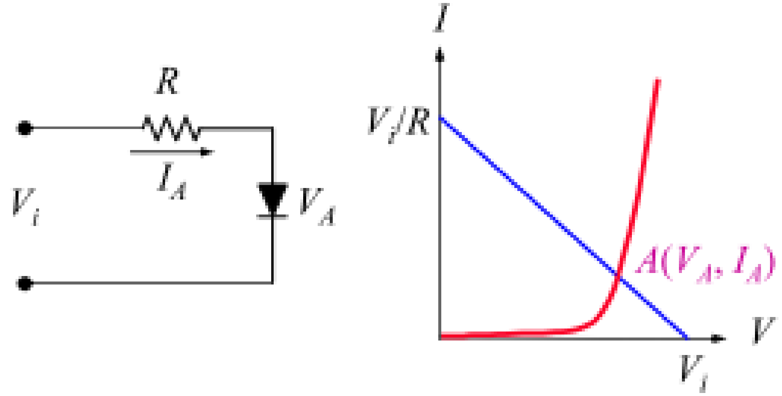
Diode Circuit

- Consider a simple diode circuit as shown. The KVL dictates:

$$V_i = I_A R + V_A$$

Then,

$$I_A = \frac{V_i}{R} - \frac{V_A}{R}$$



The above equation defines a *load line*. This line intersects the *characteristic line* of diode at the *operating point A*, which determines (V_A and I_A).

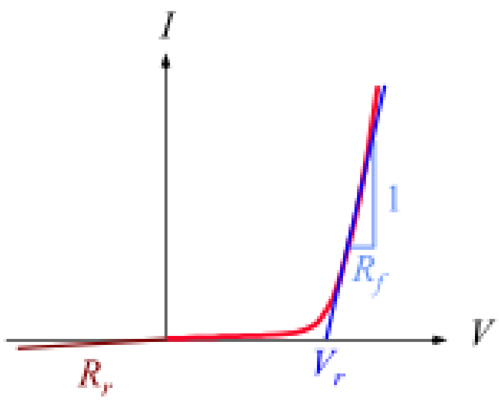
- A resistor R is often included in a diode circuit to limit the current, avoiding blow down the element.

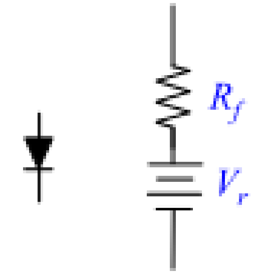
Diode Model

- The exponential trend of characteristic curve hinders algebraic calculation. The curve can be modeled as a *slant*,

$$V = V_r + I R_f$$

  where R_f is the slope of the slant. As the forward bias is greater than V_r, the diode is turned on; otherwise in off state. The value of R_f for Si is among 5 to 50 Ω.
- The equivalent circuit of a diode is revealed in the right. The model consists of a resistance and a cut-in voltage source.
- The *reverse* bias can also be modeled as a slant of slope R_r in the order of $O(10^4)$ Ω.

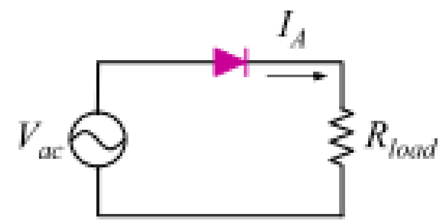


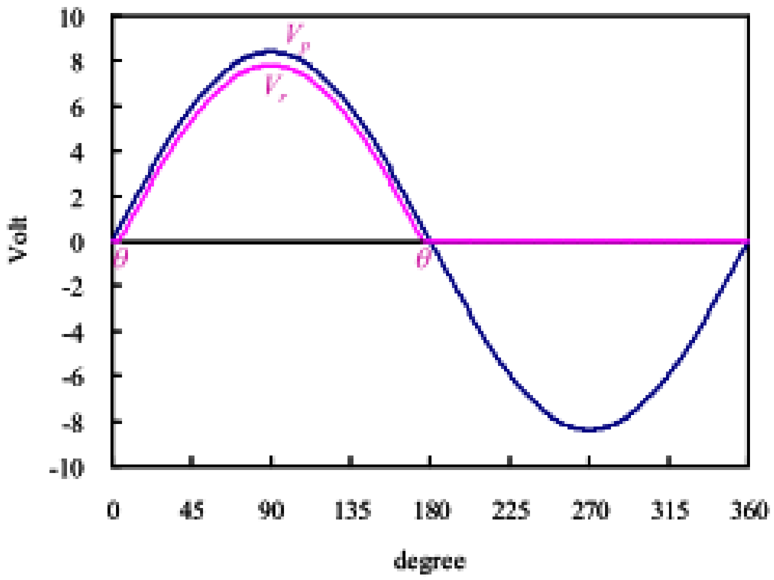


Half-wave Rectifier

- An ac source drives a resistor as shown. The voltage in the resistor is a simple sine wave.
- When a diode is placed in series in the circuit, only the positive voltage provides a forward bias to the diode, resulting in a half-wave rectifier.
- The rectified voltage shows V_r lower than the input, with a *cut-in angle* and an *extinction angle* at the rising and ceasing points,

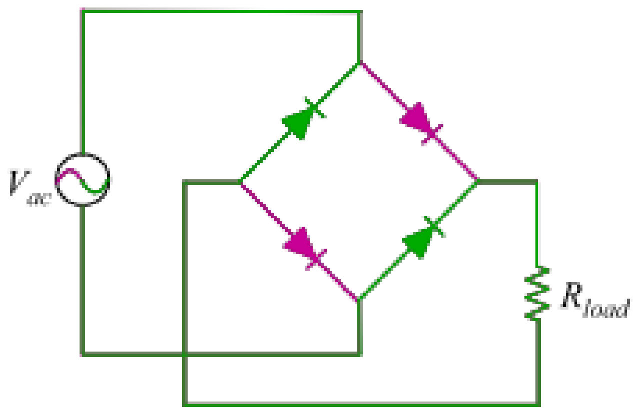
$$\theta = \sin^{-1}(V_r/V_p)$$

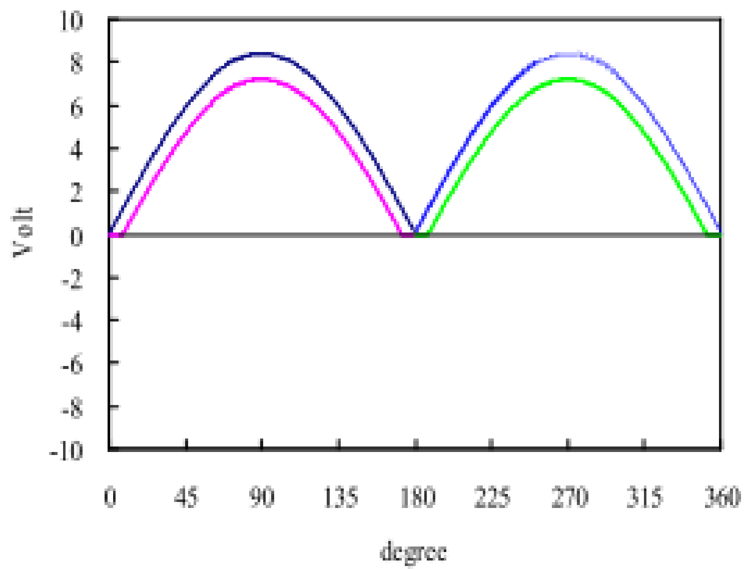




Full-wave Rectifier

- The *bridge rectifier* is constructed by four diode to fully utilize both half waves. Each half wave passes through two diodes, resulting in $2V_r$ voltage drop. The load duty for each diode is half of the cycle.





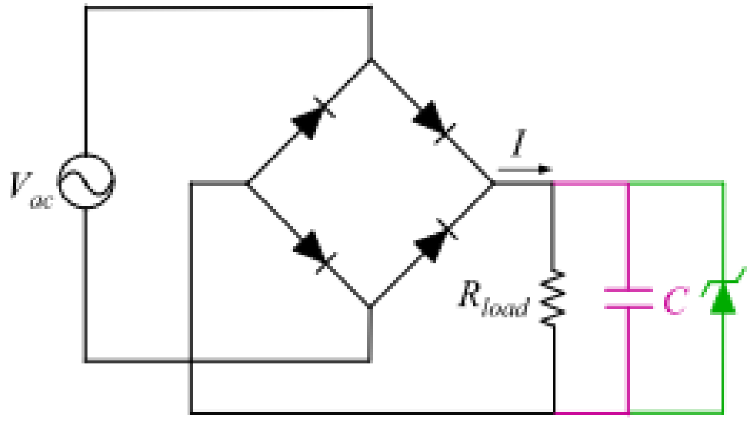
Ripple

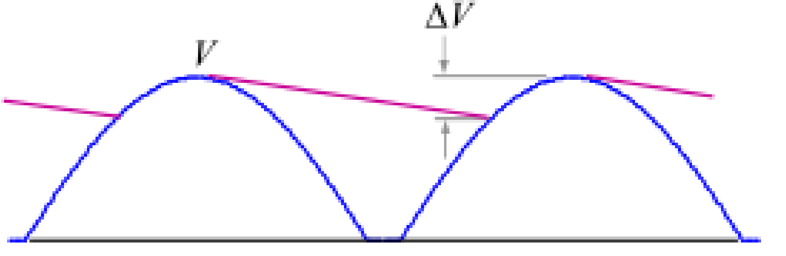
The rectified waves is far away from the desired dc source. A capacitor can be parallelized to the load to reduce the fluctuation. The voltage of the *ripple* can be approximated by the equation. A Zener diode or voltage regulator can cut the voltage to a flat level.

half wave $\Delta V = \dfrac{V}{fRC} = \dfrac{I}{fC}$

full wave $\Delta V = \dfrac{V}{2fRC} = \dfrac{I}{2fC}$

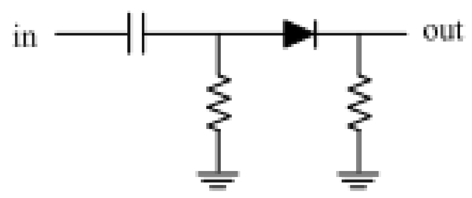
$RC >> 1/f$



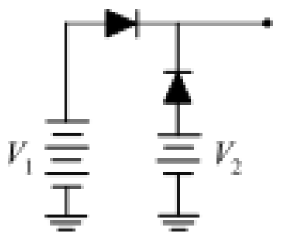


Applications of Diode

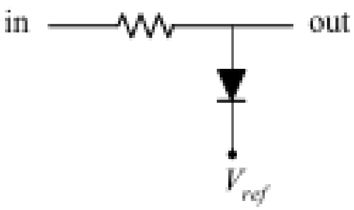
- Signal rectifier: only allow positive (or negative) to pass with a cost of dropping voltage level V_r.



- Diode gates: turn on the other circuit (such as voltage backup) when the voltage of a normal circuit is below the level.

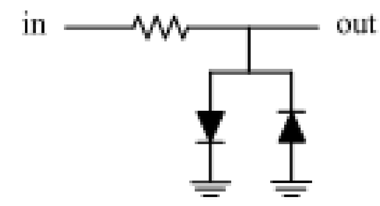


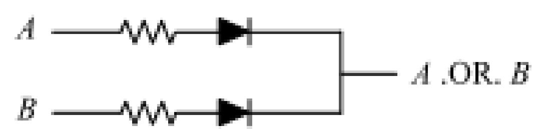
- Diode clamps: limit the range of a signal to avoid voltage exceeding certain levels. Static electric discharging is a typical example.



Applications of Diode

- Diode limiter: limits the output swing to prevent voltage saturation that might occur in the circuit of the next stage.
- Logic circuits: provide alternatives for logic circuits of fast response.





運算放大器

Feedback

- Negative feedback
  - Stabilizes the signal.
  - Lowers the gain.
  - Improves linearity and frequency response.
  - Less dependent on the characteristics of the open-loop (no-feedback) amplifier and dependent only on the properties of the feedback network itself.
- Positive feedback
  - Oscillation

Operational amplifiers

- Common architecture
  - Non-inverting/inverting input
  - Power supply
  - Voltage gain
  - Output
  - Offset null

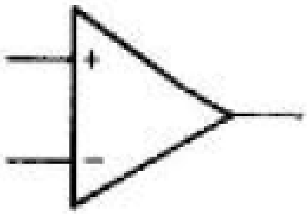


Figure 4.1

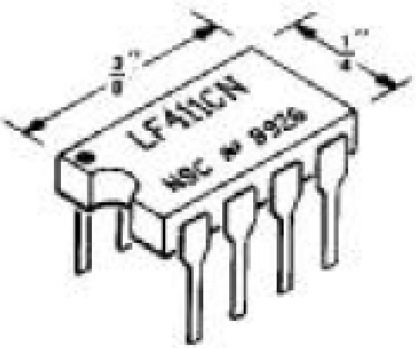


Figure 4.2. Mini-DIP integrated circuit.

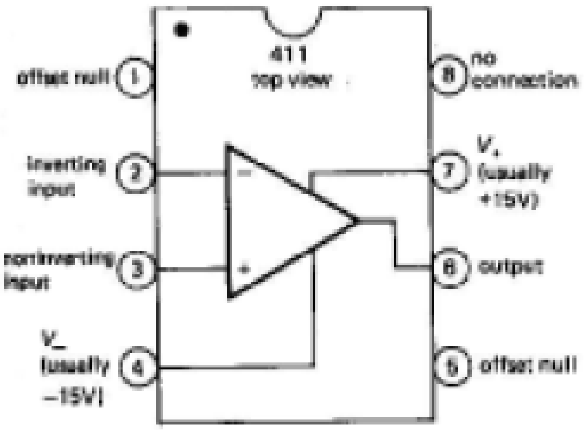


Figure 4.3

Schematic of the 741 op-amp

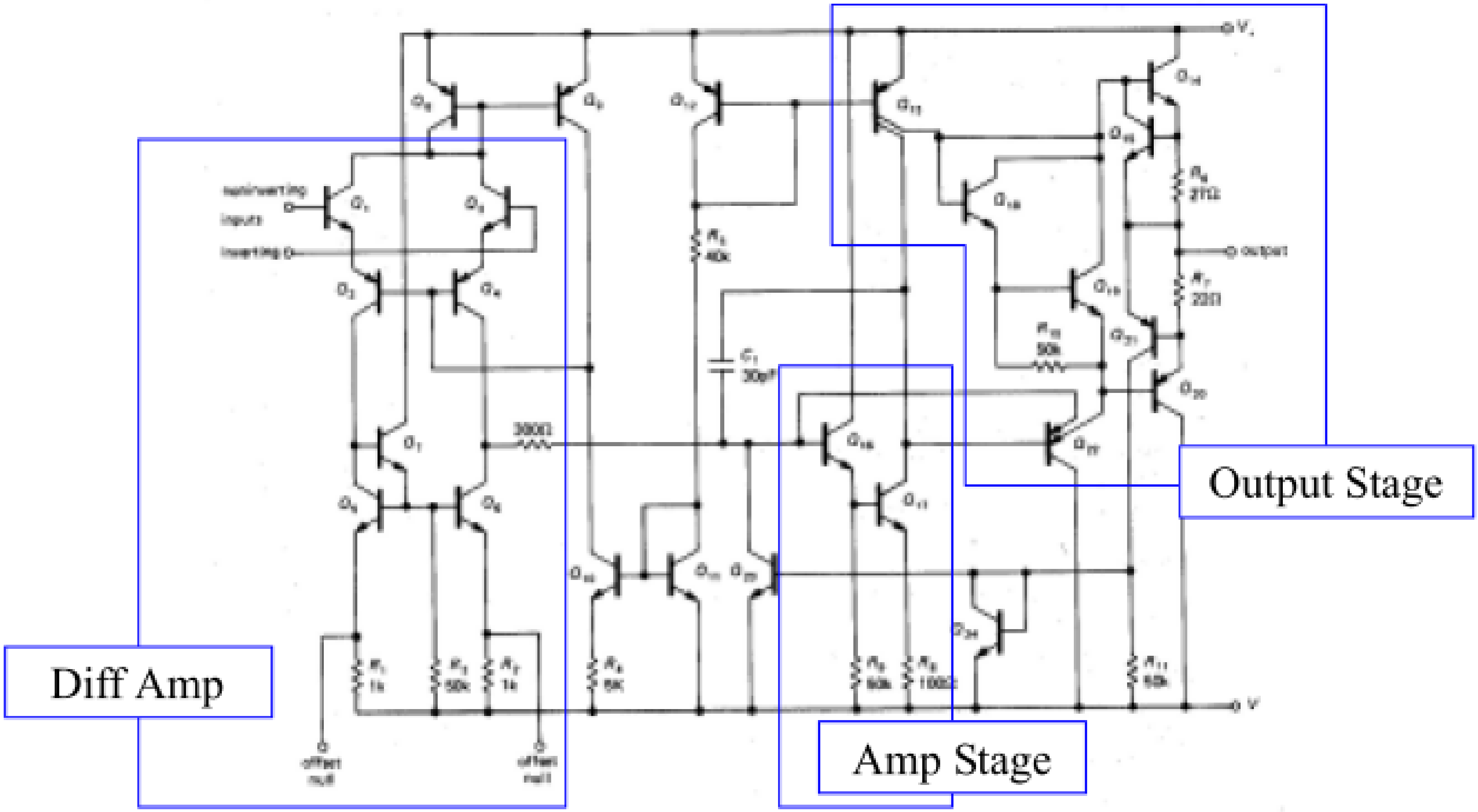


Figure 4.28. Schematic of the 741 op-amp. (Courtesy of Fairchild Camera and Instrument Corp.)

The golden rules

- Rule 1: With negative feedback, the output attempts to do whatever is necessary to make the voltage difference between the inputs zero.
  - The op-amp voltage gain is so high that a fraction of a millivolt between the input terminals will swing the output over its full range.
- Rule 2: The inputs draw no current.
  - Actually 0.2 nA for an LF411 op-amp.

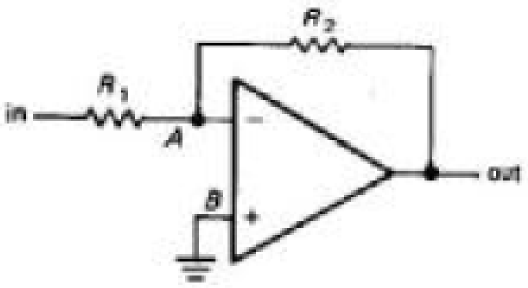


Figure 4.4. Inverting amplifier.

Inverting/noninverting amplifier

- Inverting amplifier
  - Rule 1: $V_A = V_B = 0$
  - Rule 2: $V_{in}/R_1 = I = (0 - V_{out})/R_2$
  - Voltage gain = $V_{out}/V_{in} = - R_2/R_1$
  - $Z_{in} = R_1$
  - $Z_{out} < 1\Omega$
- Noninverting amplifier
  - $V_{in} = V_A = V_{out}R_1/(R_1 + R_2)$
  - Voltage gain = $V_{out}/V_{in} = 1 + R_2/R_1$
  - $Z_{in} \sim$ infinite
  - $Z_{out} < 1\Omega$

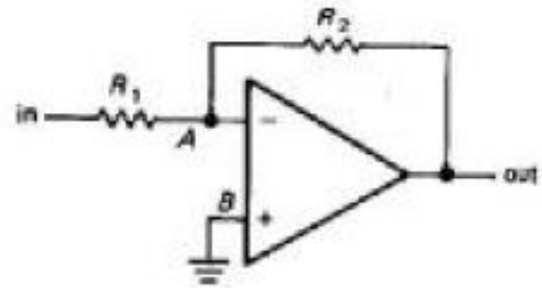


Figure 4.4. Inverting amplifier.

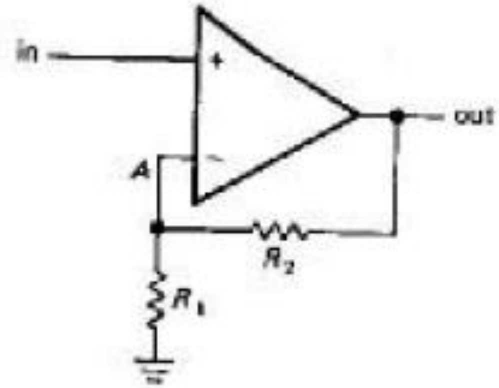


Figure 4.5. Noninverting amplifier.

AC amplifier and follower

- AC amplifier
  - Case 1: gain = 10; 3dB @16Hz.
  - Case 2: gain = 10; 3dB @17Hz.
- Follower
  - A noninverting amplifier with R_1 = infinite and $R_2 = 0$; gain = 1.

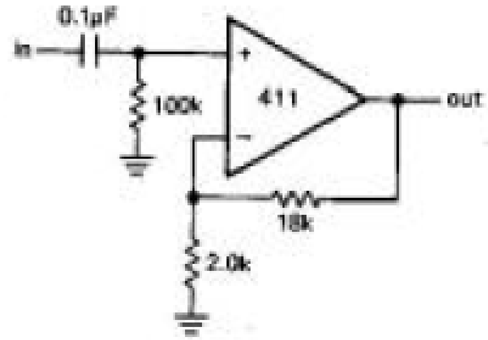


Figure 4.6

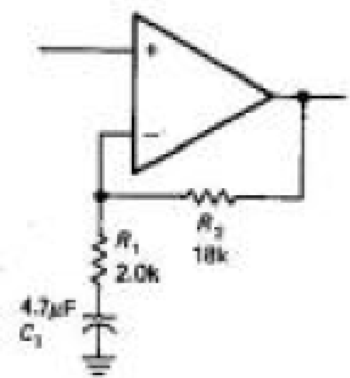


Figure 4.7

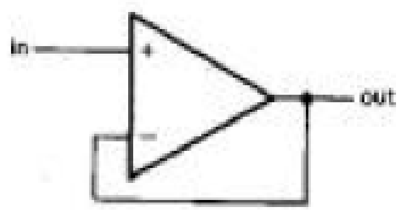


Figure 4.8. Follower.

Example circuits (2)

- Ideal current-to-voltage converter
  - Case 1
  - Case 2
- Differential amplifier
  - $(V_{out}R_1 + V_1R_2)/(R_1 + R_2) = V_2R_2/(R_1 + R_2)$

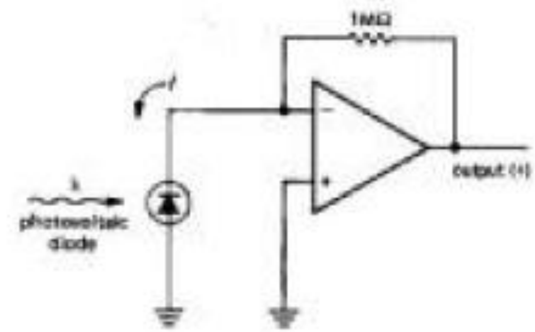


Figure 4.16

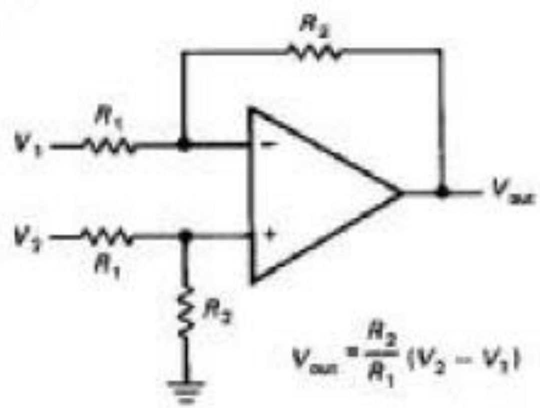


Figure 4.18. Classic differential amplifier.

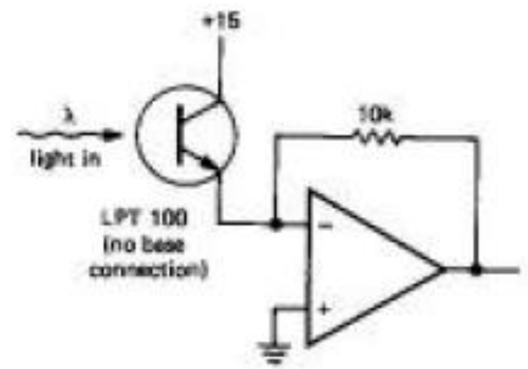


Figure 4.17

Example circuits (3)

- Summing amplifier
  - $V_{out} = -RI = -R(V_1/R + V_2/R + V_3/R)$
    $= V_1 + V_2 + V_3$

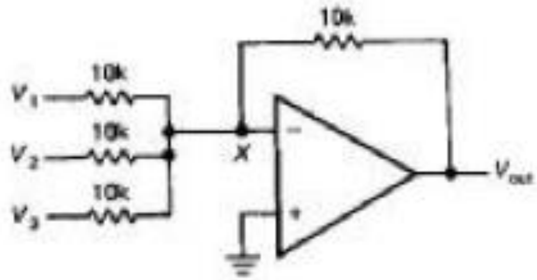
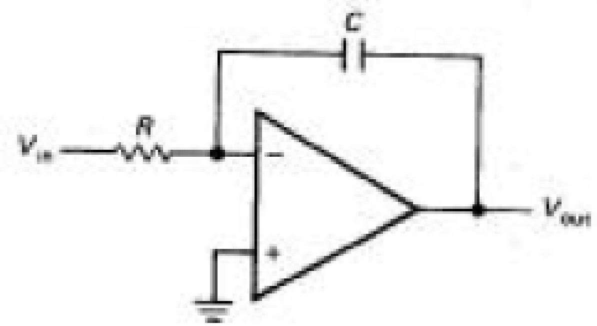


Figure 4.19

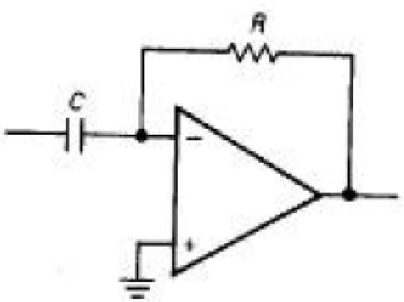
Integrators

- Integrators
  - $V_{in}/R = -C(dV_{out}/dt)$



Differentiators

- Differentiators
  - $I = C(dV_{in}/dt)$



Optional: # Comparators

- To know which of two signals is larger or to know when a given signal exceeds a predetermined value (digital: yes or no).
- Output stage: push-pull in ordinary Ops while open-collector (open-drain) in comparators and needs “pull-up resistor”.
- Some points to remember:
  - (a) Because there is no negative feedback, golden rule I is not obeyed. The inputs are not at the same voltage
  - (b) The absence of negative feedback means that the (differential) input impedance isn’t bootstrapped to the high values characteristic of op-amp circuits. If the driving impedance is too high, strange things may happen.
  - (c) Some comparators permit only limited differential input swings, as little as ±5V volts in some cases. Check the specs!

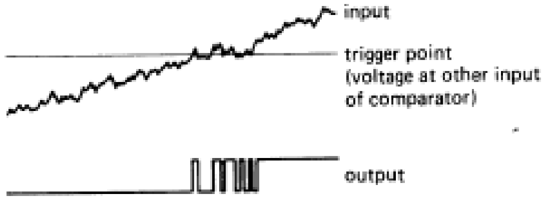
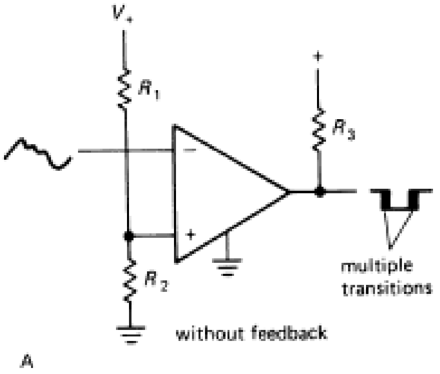
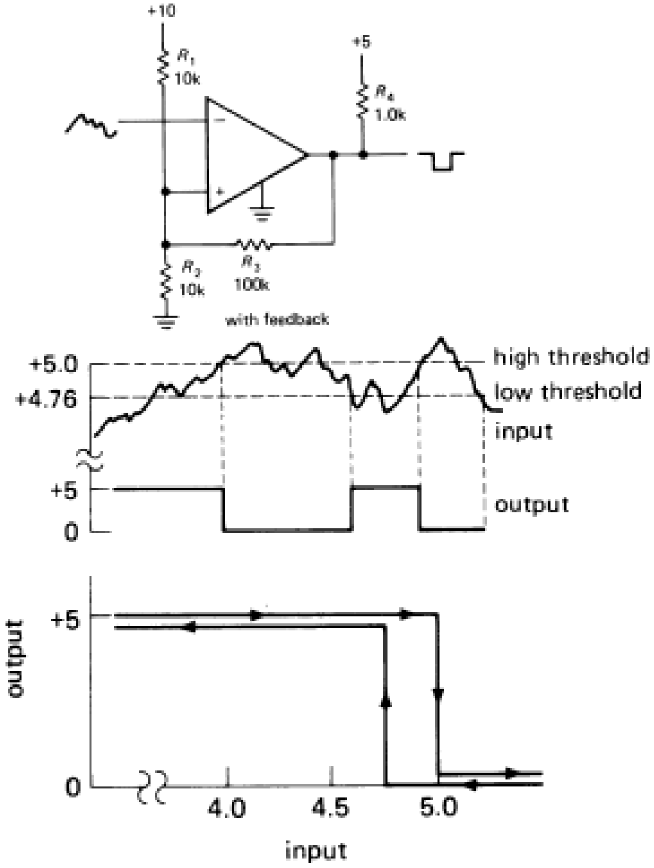


Figure 4.61



Optional: Schmitt Trigger

- For a very slowly varying input, the output swing can be rather slow.
- Worse still, if the input is noisy, the output may make several transitions as the input passes through the threshold.
- With positive feedback, the effect of R_3 is to make the circuit have two thresholds, depending on the output state (hysteresis).
- Furthermore, the positive feedback ensures a rapid output transition regardless of the speed of the input waveform.
- The hysteresis equals the output swing, attenuated by a resistive divider formed by R_1 and $R_2 \,||\, R_3$.



Optional: Derivation of Schmitt Trigger

- When $V_- < V_+$, the output transistor turns off. $V_{out} = 5V$, $V_+ = 5V$.
  $\rightarrow V_{out} = 5V$ if $V_{in} < 5V$.

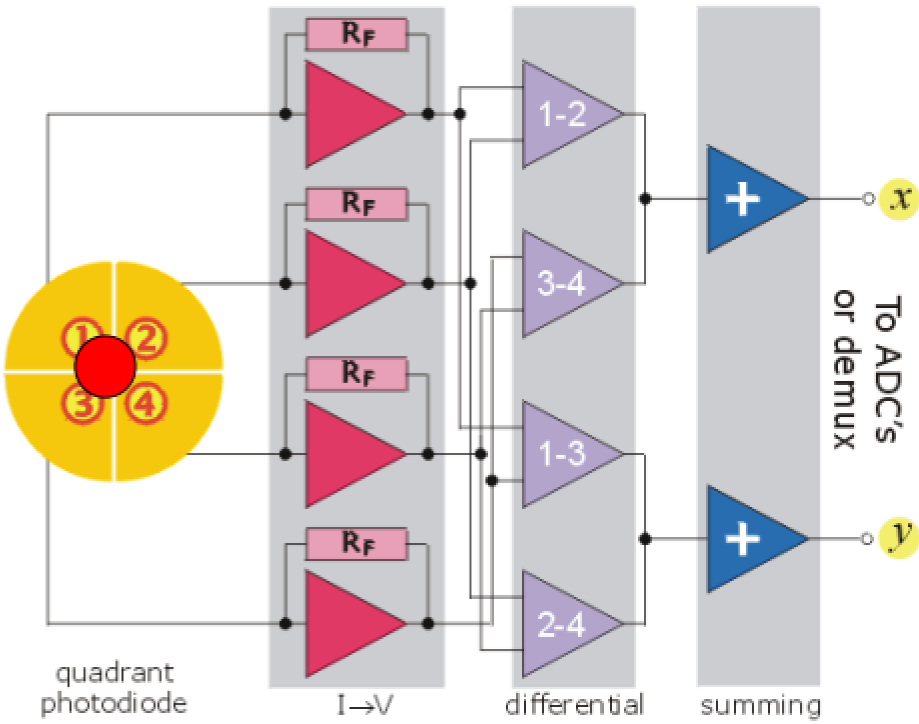
- When $V_- > V_+$, the output transistor turns on. $V_{out} = 0V$,
  $V_+ = 10V$ * a resistive divider formed by R_1 and $R_2 \| R_3$
  $= 10V*9.09/(10+9.09) = 4.76V$
  $\rightarrow V_{out} = 0V$ if $V_{in} > 4.76V$.

- If $R_1 = R_2$, the hysteresis approximates $10V * (R_1/R_3)/4 = 0.25V$.

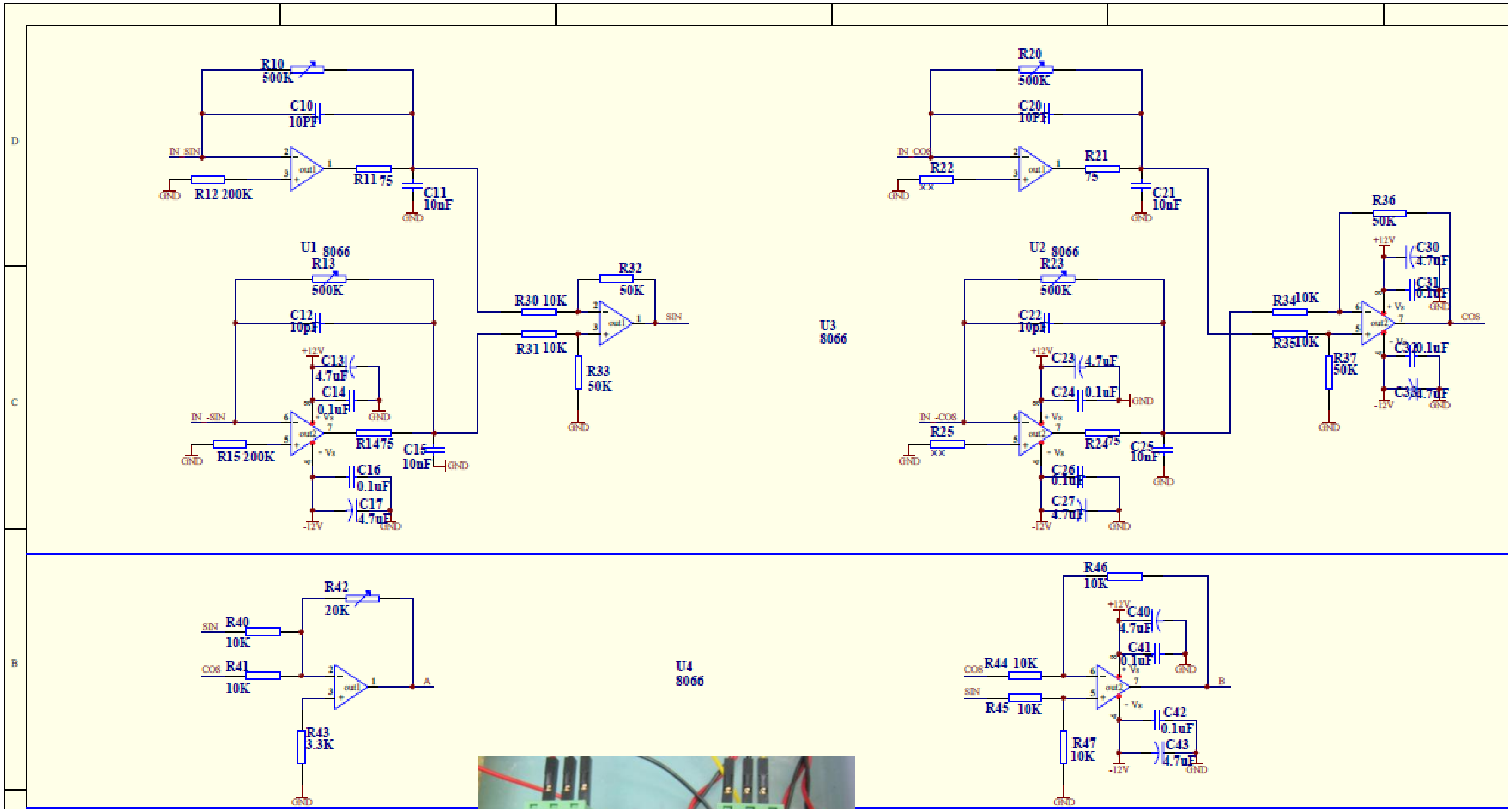
四象限感測器電路

$$\Delta X = \frac{(V_1+V_2)-(V_3+V_4)}{V_1+V_2+V_3+V_4}$$

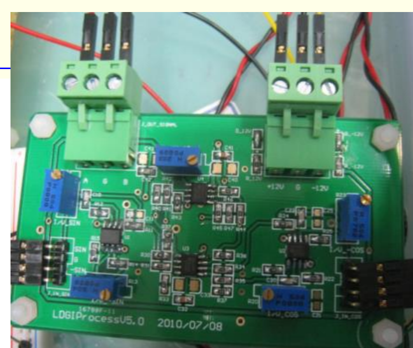
$$\Delta Y = \frac{(V_2+V_4)-(V_1+V3)}{V_1+V_2+V_3+V_4}$$



光學尺弦波處理電路



R10 500K
C10 10PF
IN SIN
R11 75
C11 10uF
R12 200K
U1 8066
R13 500K
C12 10pF
C13 4.7uF
C14 0.1uF
IN -SIN
R14 75
C15 10nF
R15 200K
C16 0.1uF
C17 4.7uF
R30 10K
R31 10K
R32 50K
R33 50K
SIN
U3 8066
R20 500K
C20 10PF
IN COS
R21 75
R22 xx
C21 10uF
U2 8066
R23 500K
C22 10pF
C23 4.7uF
C24 0.1uF
IN -COS
R24 75
R25 xx
C25 10nF
C26 0.1uF
C27 4.7uF
R34 10K
R35 10K
R36 50K
R37 50K
C30 4.7uF
C31 0.1uF
C32 0.1uF
C33 4.7uF
COS
R40 10K
R41 10K
R42 20K
R43 3.3K
A
U4 8066
R44 10K
R45 10K
R46 10K
R47 10K
C40 4.7uF
C41 0.1uF
C42 0.1uF
C43 4.7uF
B
GND
+12V
-12V



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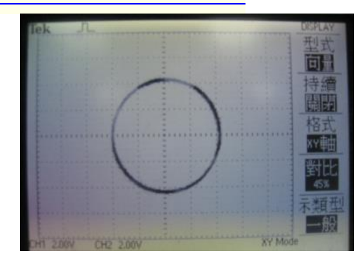
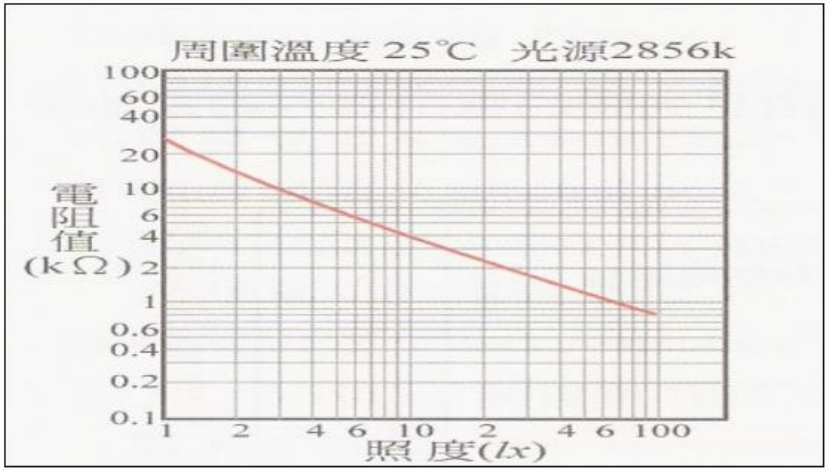
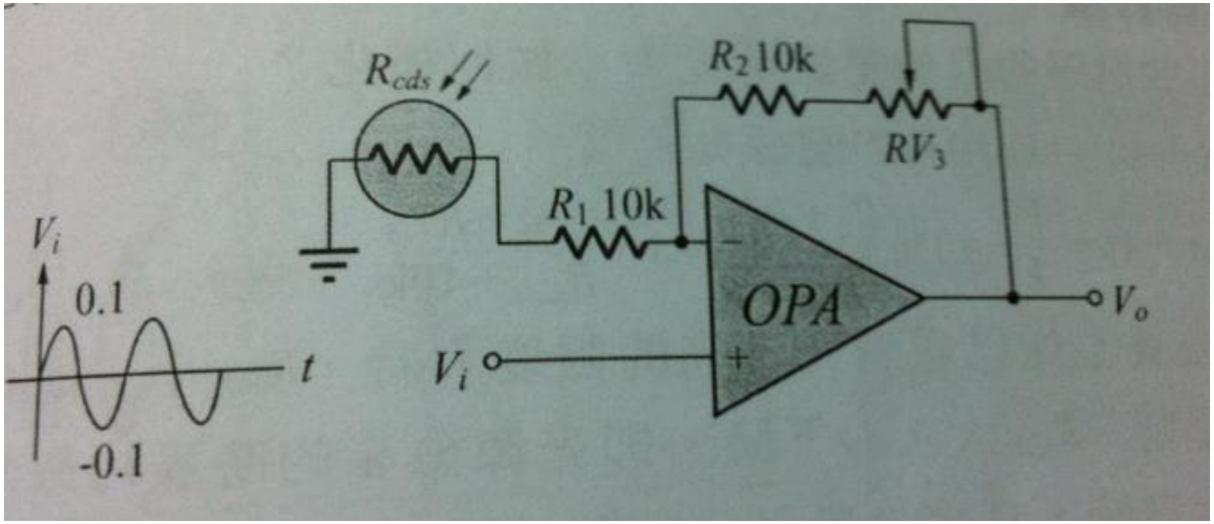


XY Mode

光敏電阻—自動調整亮度電路





Optional: 白金感測電阻溫度測量