真圓度之精密量測

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Courtesy to Prof. Kuang-Chao Fan's lecture note

何謂真圓度

- 以失圓尺寸大小表示
- 圓形工件之輪廓形狀與理想形狀偏差量
- 二個能包絡圓形工件輪廓形狀的同心圓之最小半徑差異

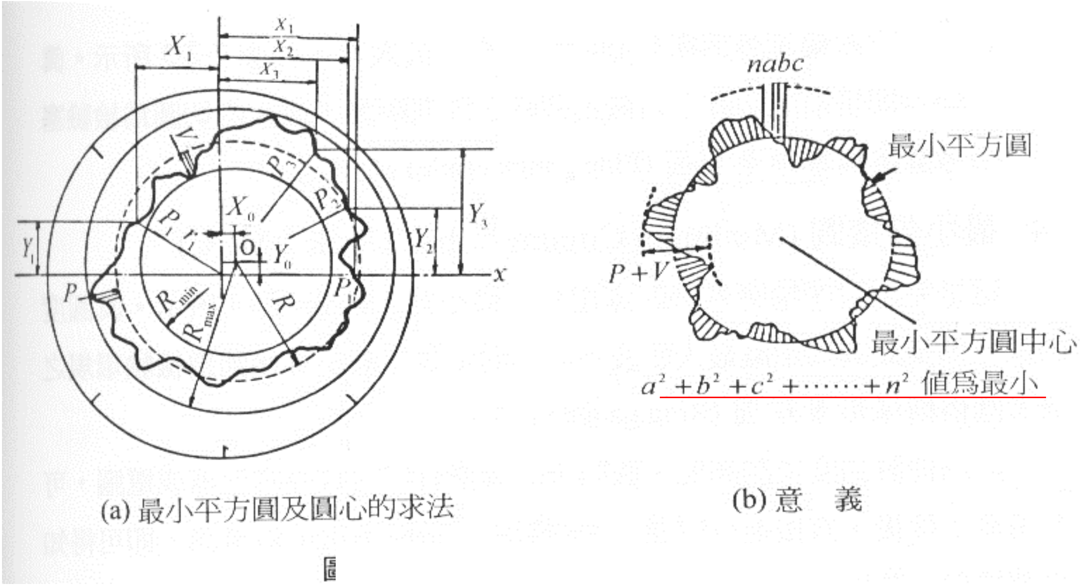
真圓度誤差評估(ANSI B89.3.1)

- 最小平方圓 (Least Squares Circle)
- 最小環帶圓(Minimum Zone Circle)
- 最大內切圓(Maximum Inscribed Circle)
- 最小外接圓(Minimum Circumscribed Circle)

最小平方圓（ LSC ）

- 最小平方圓距輪廓形狀之徑向距離之平方和為最小。
- 待測工件圓斷面之失圓等於最小平方圓其偏差值的大小失圓為其最大波峰(P)至最大波谷(V)的和(P+V)。

最小平方圓（ LSC ）



Fitting a Circle to 2D Points

Given a set of points $\{(x_i, y_i)\}_{i=1}^m$, $m \geq 3$, fit them with a circle $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the circle center and r is the circle radius. An assumption of this algorithm is that not all the points are collinear. The energy function to be minimized is

$$E(a,b,r) = \sum_{i=1}^{m} (L_i - r)^2$$

where $L_i = \sqrt{(x_i - a)^2 + (y_i - b)^2}$. Take the partial derivative with respect to r to obtain

$$\frac{\partial E}{\partial r} = -2 \sum_{i=1}^{m} (L_i - r).$$

Courtesy to David Eberly for his work “Least Squares Fitting of Data”.

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Fitting a Circle to 2D Points (cont.)

Setting equal to zero yields

$$r = \frac{1}{m}\sum_{i=1} L_i.$$

Take the partial derivative with respect to a to obtain

$$\frac{\partial E}{\partial a} = -2\sum_{i=1}^{m}(L_i - r)\frac{\partial L_i}{\partial a} = 2\sum_{i=1}^{m}\left((x_i - a) + r\frac{\partial L_i}{\partial a}\right)$$

Fitting a Circle to 2D Points (cont.)

and take the partial derivative with respect to b to obtain

$$\frac{\partial E}{\partial b} = -2\sum_{i=1}^{m}(L_i - r)\frac{\partial L_i}{\partial b} = 2\sum_{i=1}^{m}\left((y_i - b) + r\frac{\partial L_i}{\partial b}\right).$$

Setting these two derivatives equal to zero yields

$$a = \frac{1}{m}\sum_{i=1}^{m} x_i + r\frac{1}{m}\sum_{i=1}^{m}\frac{\partial L_i}{\partial a}$$

$$b = \frac{1}{m}\sum_{i=1}^{m} y_i + r\frac{1}{m}\sum_{i=1}^{m}\frac{\partial L_i}{\partial b}.$$

Fitting a Circle to 2D Points (cont.)

Replacing r by its equivalent from $\partial E/\partial r = 0$ and using $\partial L_i/\partial a = (a - x_i)/L_i$ and $\partial L_i/\partial b = (b - y_i)/L_i$, we get two nonlinear equations in a and b:

$$a = \bar{x} + \bar{L}\bar{L}_a =: F(a,b)$$

$$b = \bar{y} + \bar{L}\bar{L}_b =: G(a,b)$$

where

$$\begin{aligned} \bar{x} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} x_i \\ \bar{y} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} y_i \\ \bar{L} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} L_i \\ \bar{L}_a &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} \tfrac{a-x_i}{L_i} \\ \bar{L}_b &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} \tfrac{b-y_i}{L_i} \end{aligned}$$

Fitting a Sphere to 3D Points

Given a set of m points {(xi, yi, zi)}, m≥4, fit them with a sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ where (a, b, c) is the sphere center and r is the sphere radius. An assumption of this algorithm is that not all the points are coplanar. The energy function to be minimized is:

$$E(a,b,c,r) = \sum_{i=1}^{m} (L_i - r)^2$$

where $L_i = \sqrt{(x_i - a)^2 + (y_i - b)^2 + (z_i - c)}$. Take the partial derivative with respect to r to obtair

$$\frac{\partial E}{\partial r} = -2\sum_{i=1}^{m} (L_i - r).$$

Fitting a Sphere to 3D Points (conti)

Setting equal to zero yields

$$r = \frac{1}{m}\sum_{i=1} L_i.$$

Take the partial derivative with respect to a to obtain

$$\frac{\partial E}{\partial a} = -2\sum_{i=1}^{m}(L_i - r)\frac{\partial L_i}{\partial a} = 2\sum_{i=1}^{m}\left((x_i - a) + r\frac{\partial L_i}{\partial a}\right)$$

to *b* to obtain

$$\frac{\partial E}{\partial b} = -2\sum_{i=1}^{m}(L_i - r)\frac{\partial L_i}{\partial b} = 2\sum_{i=1}^{m}\left((y_i - b) + r\frac{\partial L_i}{\partial b}\right)$$

to *c* to obtain

$$\frac{\partial E}{\partial c} = -2\sum_{i=1}^{m}(L_i - r)\frac{\partial L_i}{\partial c} = 2\sum_{i=1}^{m}\left((z_i - c) + r\frac{\partial L_i}{\partial c}\right)$$

Fitting a Sphere to 3D Points (conti)

Setting these three derivatives equal to zero yields

$$a = \frac{1}{m}\sum_{i=1}^{m} x_i + r\frac{1}{m}\sum_{i=1}^{m}\frac{\partial L_i}{\partial a}$$

$$b = \frac{1}{m}\sum_{i=1}^{m} y_i + r\frac{1}{m}\sum_{i=1}^{m}\frac{\partial L_i}{\partial b}.$$

$$c = \frac{1}{m}\sum_{i=1}^{m} z_i + r\frac{1}{m}\sum_{i=1}^{m}\frac{\partial L_i}{\partial c}.$$

Fitting a Sphere to 3D Points (conti)

Replacing r by its equivalent from $\partial E/\partial r = 0$ and using $\partial L_i/\partial a = (a - x_i)/L_i$, $\partial L_i/\partial b = (b - y_i)/L_i$, and $\partial L_i/\partial c = (c - z_i)/L_i$, we get three nonlinear equations in a, b, and c:

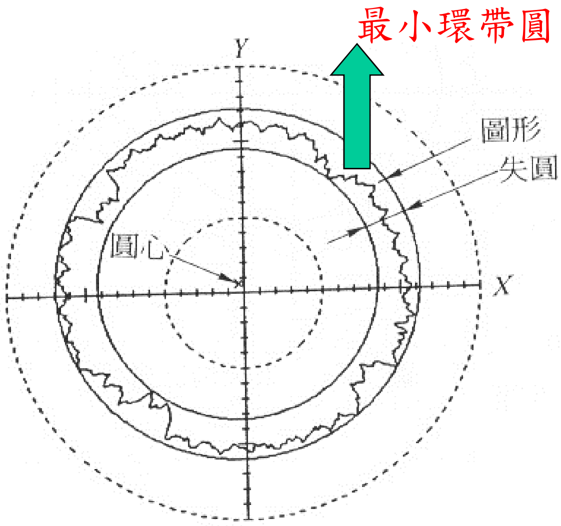
$$a = \bar{x} + \bar{L}\bar{L}_a =: F(a,b,c)$$
$$b = \bar{y} + \bar{L}\bar{L}_b =: G(a,b,c)$$
$$c = \bar{z} + \bar{L}\bar{L}_c =: H(a,b,c)$$

$$\begin{aligned}
\bar{x} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} x_i \\
\bar{y} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} y_i \\
\bar{z} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} z_i \\
\bar{L} &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} L_i \\
\bar{L}_a &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} \frac{a-x_i}{L_i} \\
\bar{L}_b &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} \frac{b-y_i}{L_i} \\
\bar{L}_c &= \tfrac{1}{m}\textstyle\sum_{i=1}^{m} \frac{c-z_i}{L_i}
\end{aligned}$$

Courtesy to David Eberly for his work “Least Squares Fitting of Data”.

最小環帶圓（ MZC ）

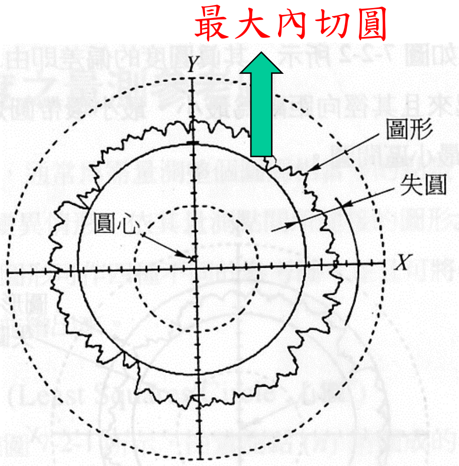
- 又稱最小區間圓
- 二個**同心圓**將輪廓形狀圓形包絡起來且其**徑向距離為最小**



MZC: Minimum zone circle

最大內切圓（ MIC ）

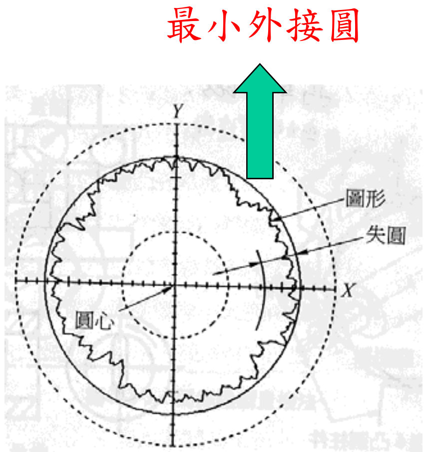
- 又稱**塞形量規圓**，只適用於**內圓**，以內接於被測圓輪廓且**半徑為最大**的內接圓。
- 完全被輪廓外形所包圍而無相交之最大圓
- 沿內切圓上的最大波峰之徑向距離
- 圓心所作包容被測圓輪廓為外圓，兩同心圓的半徑差即為圓度誤差。



(Maximum Inscribed Circle, MIC)

最小外接圓（ MCC ）

- 又稱環形量規圓
- 完全封閉輪廓外形最小圓



Minimum circumscribed circle (*MCC*)

Minimum Zone

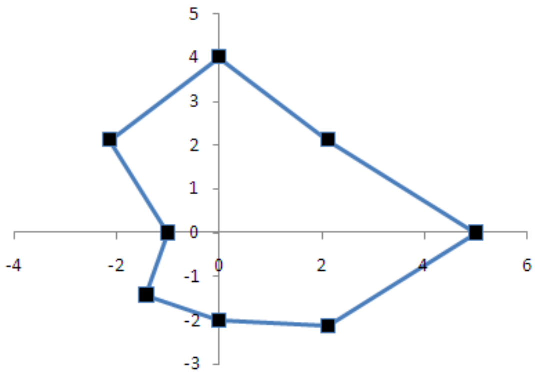
方法:

1. 先找 最小外接圓 的三控制點
2. 再找 最大內切圓 的三控制點
3. MZ可從上述 6點中找出 2-2控制點, 且須符合 2—2 原則

Home Work

Find the Minimum Zone Solution of the following circular data

半徑	角度
5	0
3	45
4	90
3	135
1	180
2	225
2	270
3	315



真圓度的量測方式

- 直徑法
- 周緣限制量規法
- 兩頂心間旋轉法
- V型塊法
- 三點探針法
- 準確主軸法

直 徑 法

- 二平面量測工具許多不同位置之直徑尺寸大小
- 真圓度即為工件最大徑與最小徑的差值

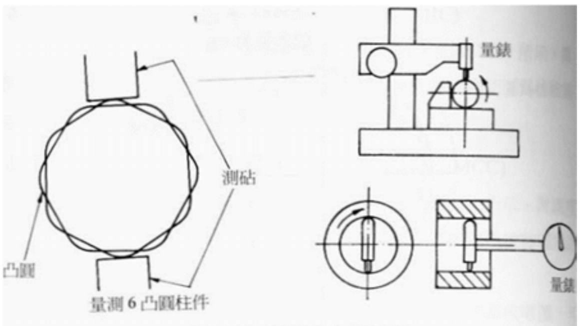
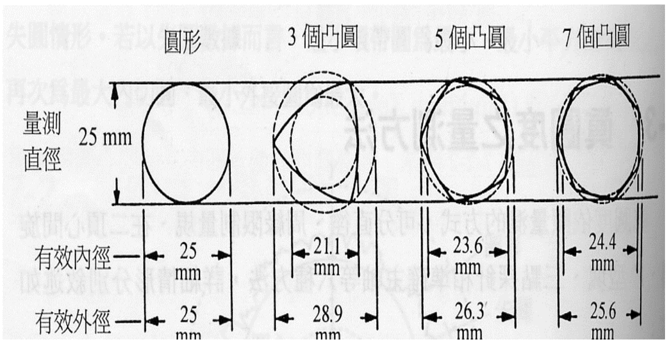


圖 7-3-1 直徑法量測眞圓度

直 徑 法

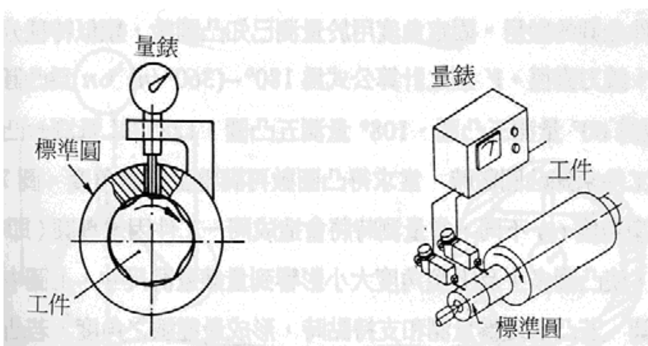
- 工件有３、５ 、 ７等凸圓其量測尺寸相同，但有效內外徑則不同
- 考慮相同直徑，失圓愈大就愈難配合
- 量測時受本身精度之影響
- 適用於橢圓形或偶數凸圓形之量測



量測尺寸相同，但有效內外徑則不同

周緣限制量規法

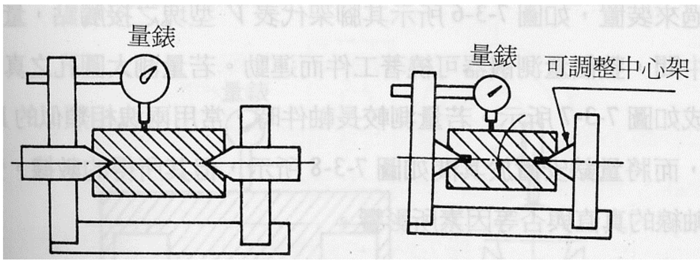
- 以一量錶來檢驗真圓度
- 無法量測其他幾何特性，如真平度、同心度
- 量測數據與待測工件形狀有關，故少用



周緣限制量規法

兩頂心間旋轉法

- 僅限用於有中心孔或具精確中心位置
- 數據含有偏位、工件曲率、圓心不完美的誤差
- 常將量錶裝置於靠近工件末端以消除其誤差



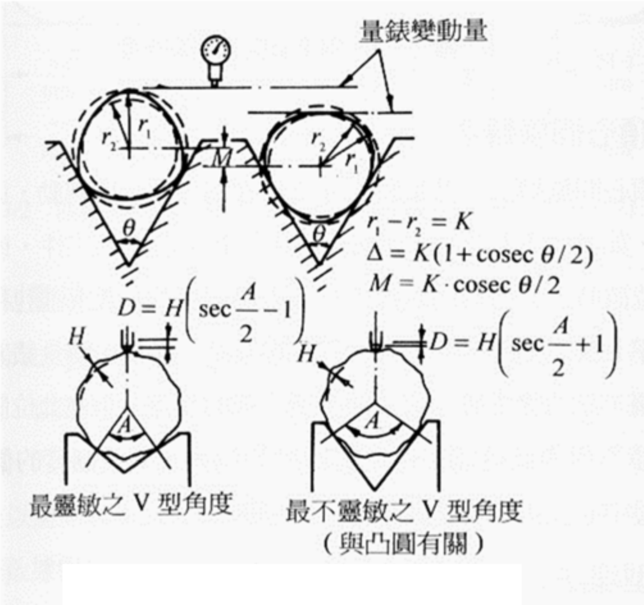
兩頂心間旋轉法

V型塊法

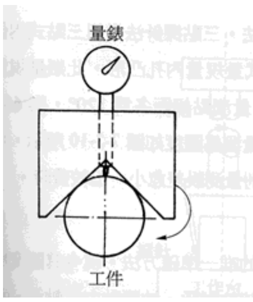
- 即三點法
- 具固定角度或可調整夾角二種
- 固定角度用於量測已知凸圓數

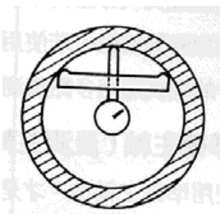
  角度= $180^\circ - (360^\circ/n)$ …n為凸圓數
- 若凸圓同為量測和支持點時，形成最靈敏的角度
- 容易使用，卻不十分準確

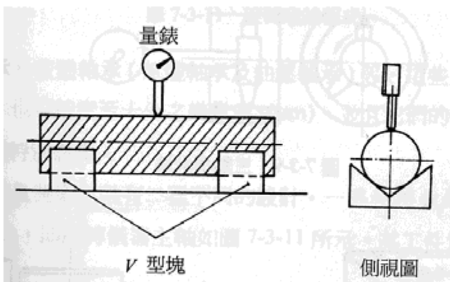
V型塊法



V型塊法



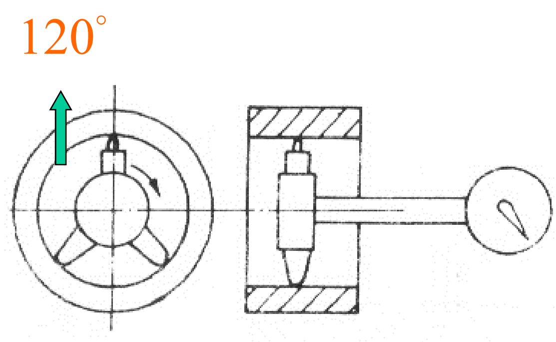




量測大軸件 量測內孔 量測長軸件

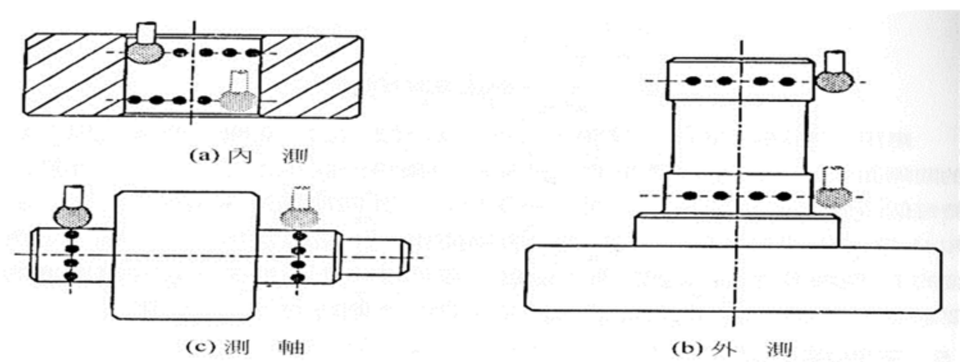
三點探針法

- 有間隔的120°三點探針
- 量測不規格幾何形狀工件，非常有效
- 常用三點式內側分厘卡作量測



三點探針法

三點探針法

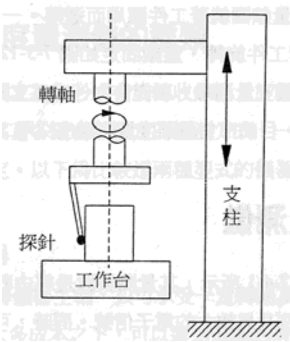


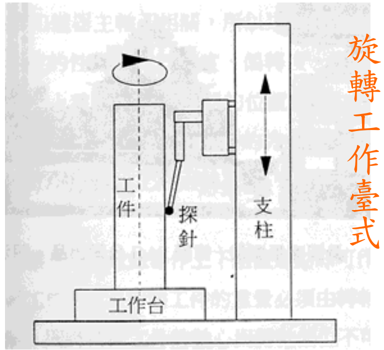
三次元量床量測軸之真圓度及圓柱度

- 依所設計的程式選用多點量測，點數愈多則量測誤差較小，但較費時

準確主軸法

旋轉收錄器式





旋轉工作臺式

- 量測真圓度的唯一準確方法
- 分為旋轉收錄器式和旋轉工作臺式

旋轉收錄器式

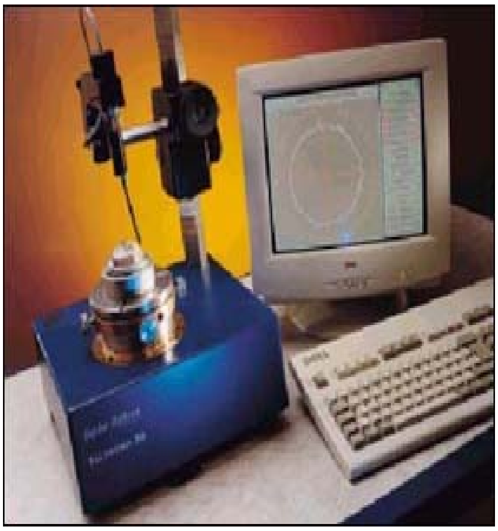
- 工件固定，主軸及量錶繞工件旋轉
- 精密的儀器主軸只承受一定轉速即固定負荷的收錄器，可達高精度
- 工作台不屬於量測系統，故工件重量不受限制

旋轉工作臺式

- 工件旋轉，量錶固定
- 使用兩支收錄器可縮短量測時間
- 收錄器和主軸不相關，容易量測其他性質
- 有更多方式決定收錄器位置，不需用長型或曲型的探針臂
- 旋轉工作台及工件重量由轉軸支持，受限制
- 轉軸與工件軸心不同時需調整，否則易有誤差

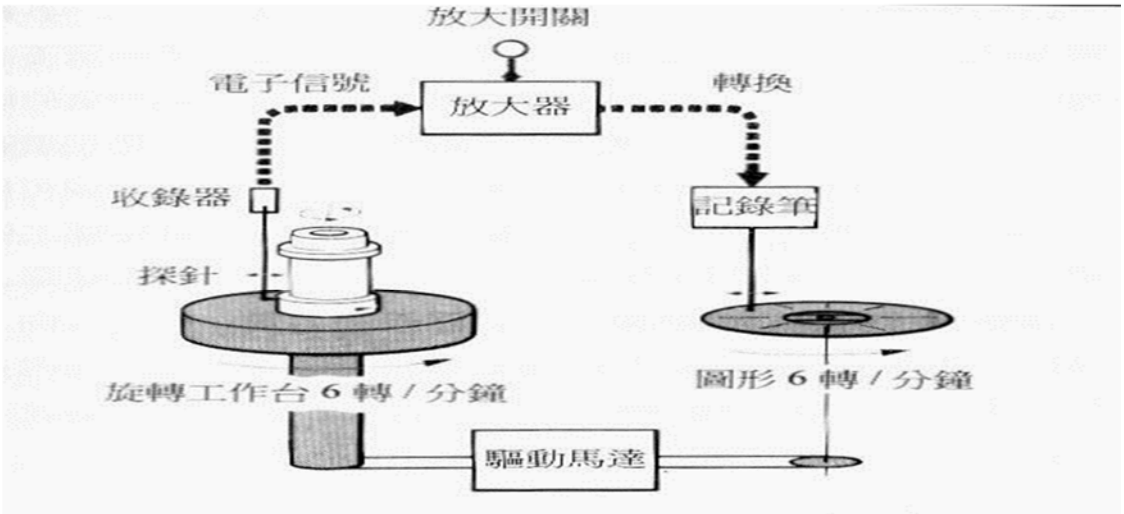
真圓度量測儀

- 經收錄器將探針的機械式運動，轉換成按比例放大的電子信號
- 再由量錶繪圖或由螢幕顯示參考圓



Talyrond的真圓度量測儀

真圓度量測儀的構造



真圓度量側儀之構造圖

真圓度量測儀的主要組成

- 工作台

  作徑向移動及傾斜角度等調整

- 收錄器

  使用線性差動變壓器 ，探針使用硬鋼而針尖使用寶石

- 紀錄器

  有極座標和線性兩種，用墨水或熱感應紀錄，現在也有用印表機節省費用

真圓度量測儀的主要組成

- 放大器

a. 將探針移動傳到收錄器而發出電子信號，將此信號放大

b. 用來過濾信號當濾波器

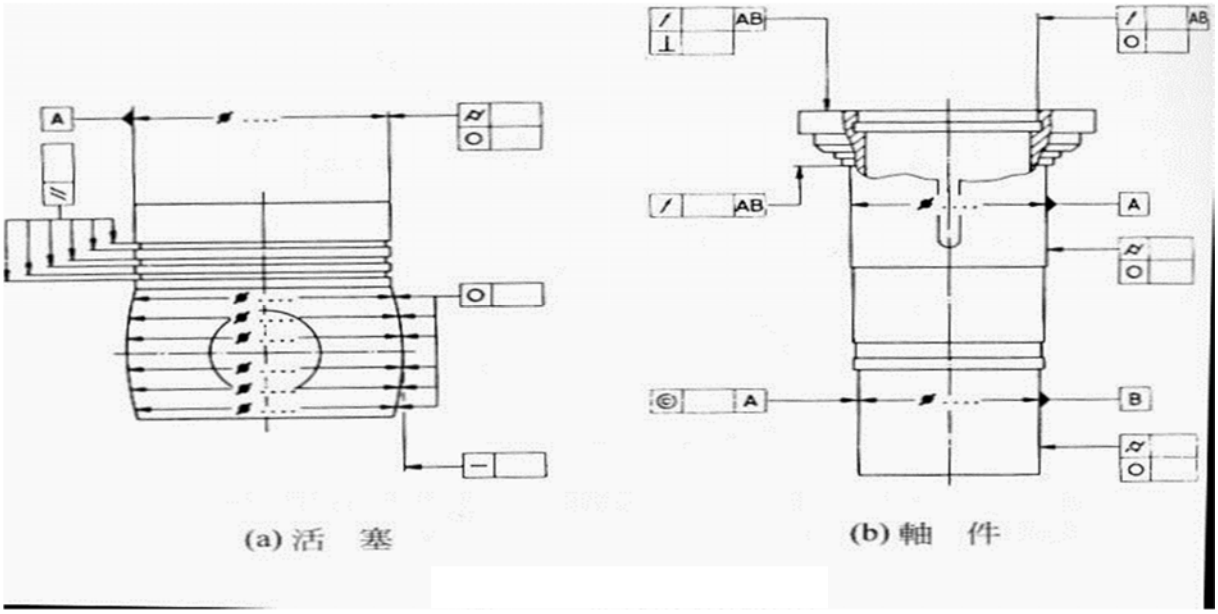
c. 一般常用低通率波為15、50、150、500等每轉反應週期數

真圓度量測儀的應用

- 作真圓度量測外，尚可作真直度、圓柱度、同心度、垂直度、平行度、圓偏轉度、總偏轉度等幾何工差的量測
- 活塞為作平行、真直、圓柱度的量測
- 軸件為量測垂直度、圓偏轉度、同心度、真直度、圓柱度

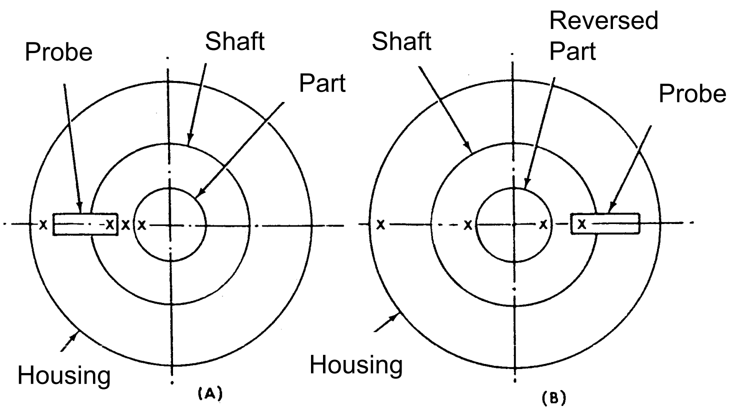
MEAS

真圓度量測儀的應用



MEAS

真圓度誤差補償



P: Part Profile

S: Spindle Run-out Error

$T1(\Theta) = P(\Theta)+S(\Theta)$ $T2(\Theta) = P(\Theta) - S(\Theta)$

$P(\Theta) = [T1(\Theta)+T2(\Theta)]/2$

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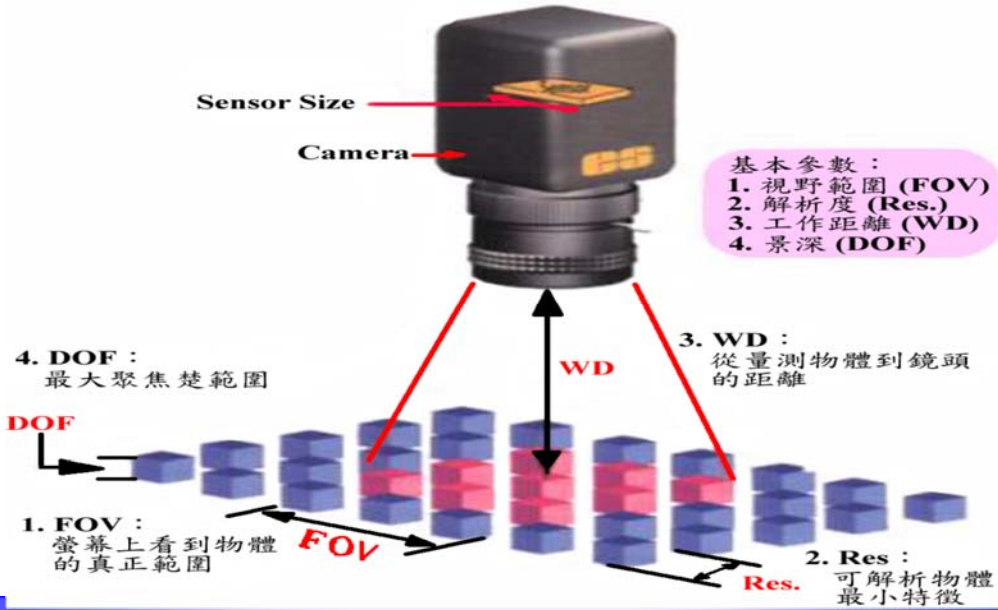
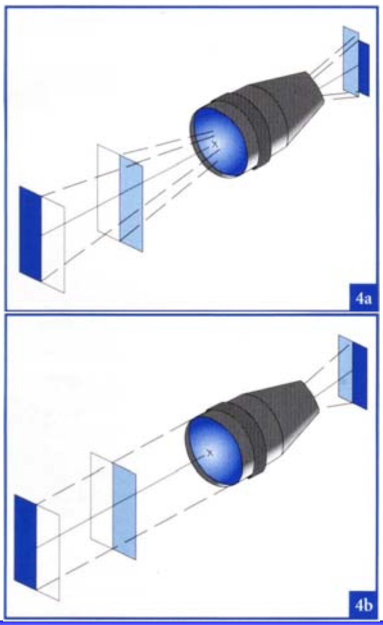
微圓度量測：光學影像放大法



Telecentric Lens 的應用



CCD參數校正方法基本概念

・應用複迴歸分析法於CCD二維座標系統之校正

二次函數進行最小迴歸分析，依上節所述原理即可得到所需之對映函式(即比例尺)。茲計算如下：

假設此二次函數形式如下：

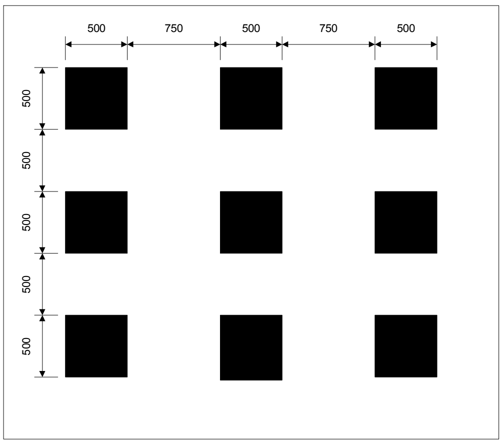
$$X = a_1u^2 + b_1uv + c_1v^2 + d_1u + e_1v + f_1$$

$$Y = a_2u^2 + b_2uv + c_2v^2 + d_2u + e_2v + f_2$$

其中，（X,Y）代表空間座標(單位：μm)；

(u,v)代表相對映之影像座標(單位：pixel)

CCD參數校正方法基本概念



校正用mask尺寸圖（範例）

經由實際校正，所得係數為：

$$a_1 = 0.00000, a_2 = 0.00000$$

$$b_1 = 0.00000, b_2 = 0.00000$$

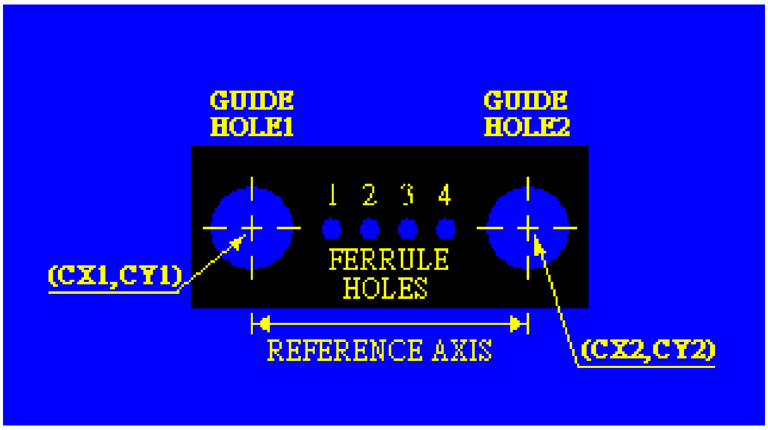
$$c_1 = 0.00001, c_2 = 0.00001$$

$$d_1 = 3.75139, d_2 = 0.03368$$

$$e_1 = -0.03506, e_2 = 3.66189$$

$$f_1 = -313.57671, f_2 = -158.32445$$

Ferrule接頭檢測綜觀



量測Guide Holes尺寸、Pitch、真圓度

量測Ferrule Holes尺寸、Pitch、真圓度、Positions(各Ferrule Holes相對左右Guide Holes圓心連線所產生之位置偏移)

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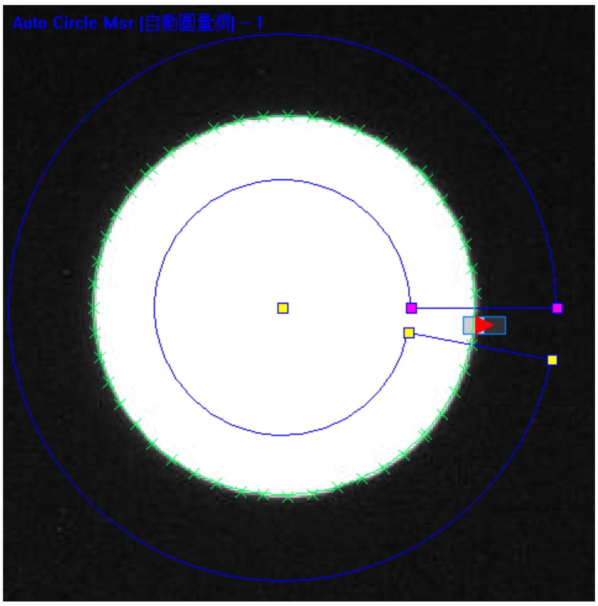
Edge Detection法則找出圓週上邊界點



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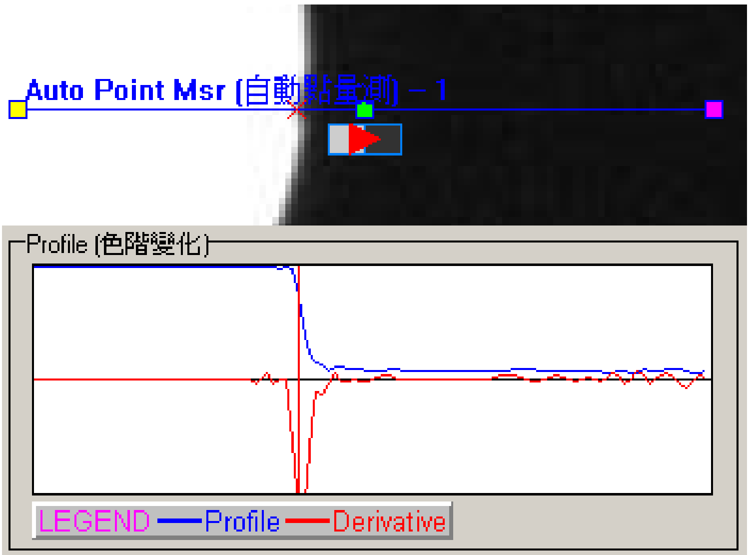
Edge Detection法則



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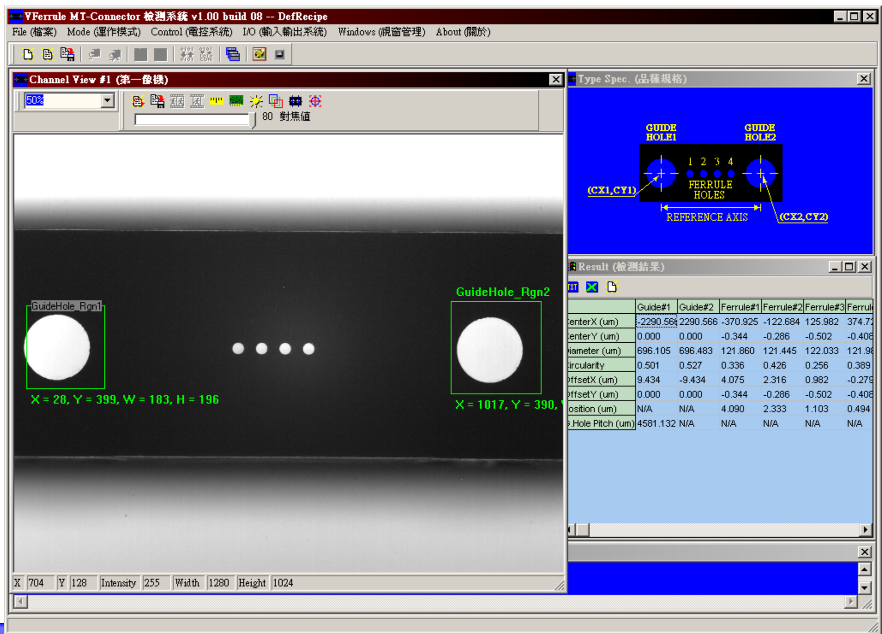
Ferrule接頭檢測



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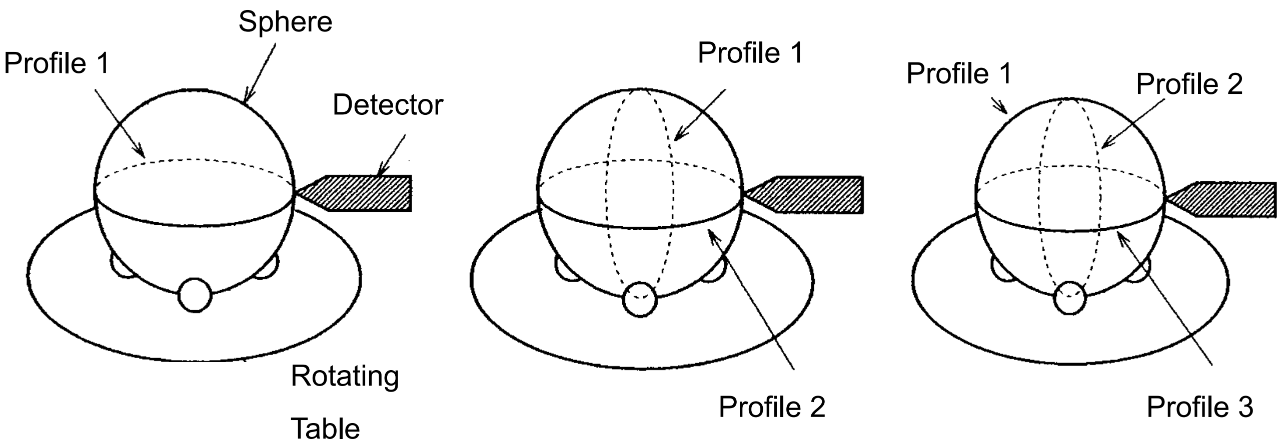
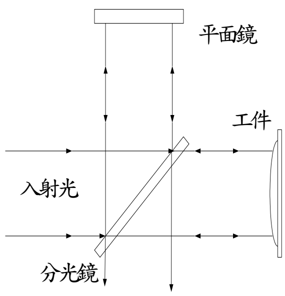
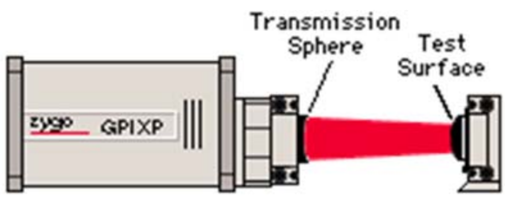
Spherical Surface Measurement ISO 3290, JIS B 1501

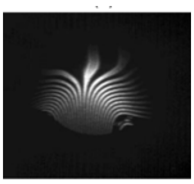


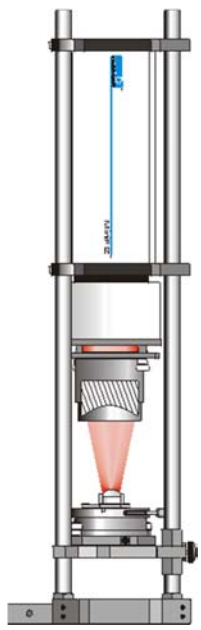
(a) First measurement (b) Second measurement (c) Third measurement

球面誤差光學干涉法

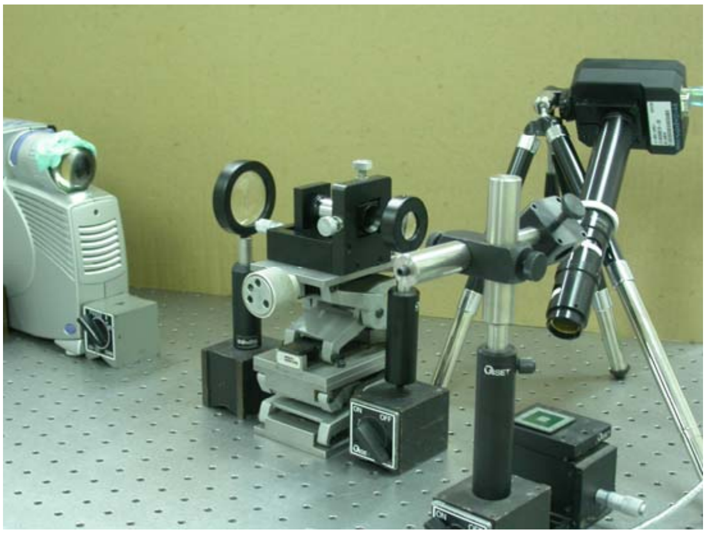


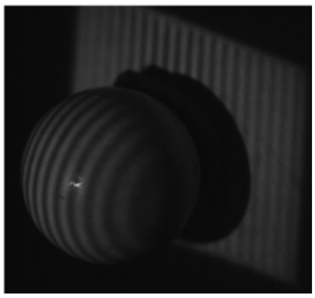


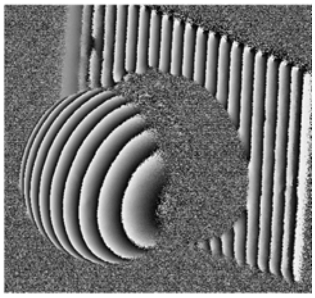




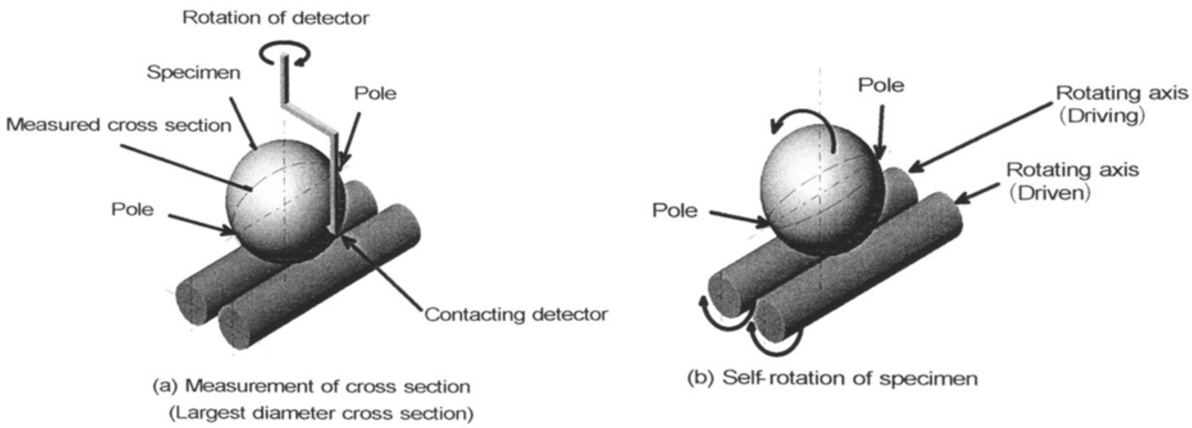
Stereo Microscope



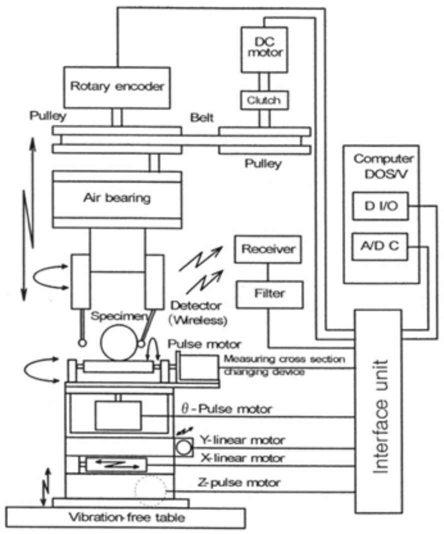




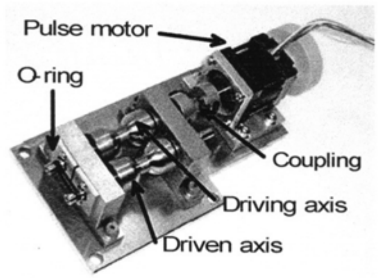
Principle of Spherical Surface Measurement (Hiroyuki Kawa, Precision Eng. 2003)

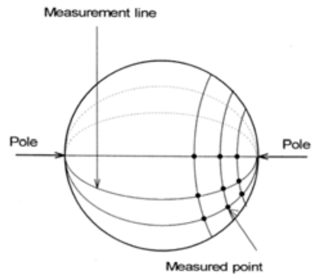


Spherical Measurement Equipment

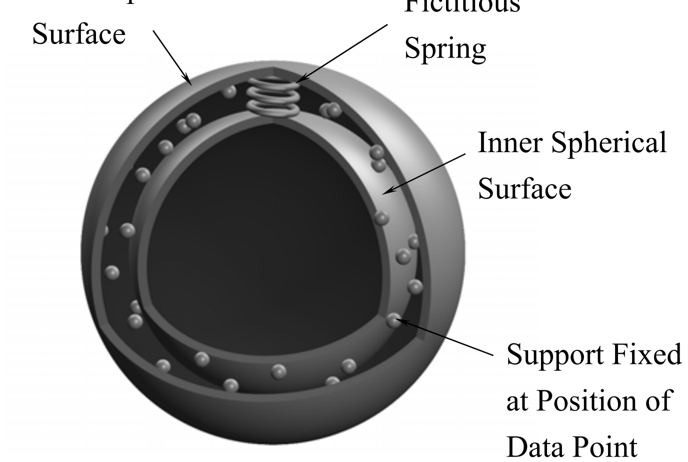


Schematic diagram of the system





ANALYSIS OF MINIMUM ZONE SPHERICITY ERROR USING MINIMUM POTENTIAL ENERGY THEORY



K.C Fan and J.C. Lee, Precision Engineering, 1999

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