Appendix

A1. Proof of Theorem 1

The cost function *J* of (7) can be expanded as follows:

$$J = \begin{bmatrix} X^T & u^T \end{bmatrix} \begin{bmatrix} R & S^T \\ S & Q \end{bmatrix} \begin{bmatrix} X \\ u \end{bmatrix} = X^T RX + u^T SX + X^T S^T u + u^T Qu. \qquad \text{(A1)}$$

Using an intermediate term $X^T S^T Q^{-1} SX$, (A1) can be rearranged as:

$$J = \left(u^T + X^T S^T Q^{-1}\right) Q \left(u + Q^{-1} SX\right) + X^T \left(R - S^T Q^{-1} S\right) X, \qquad \text{(A2)}$$

where the second term of (A2) is independent of control *u*, while the first term is always greater than zero because *Q* is positive definite ($Q>0$). That is,

$$\left(u^T + X^T S^T Q^{-1}\right) Q \left(u + Q^{-1} SX\right) \geq 0.$$

Therefore, the minimum cost can be achieved by setting $u = -Q^{-1} SX$ such that the first term is zero and the minimum cost is:

$$\min_u J = X^T \left(R - S^T Q^{-1} S\right) X .$$

A2. Dynamics of the 2D STS model

Refer to the 2D STS model of Fig. A1.

(a). Setting point A as the origin of the coordinate system X and Y, the positions of the CoMs of the leg, thigh and trunk can be represented as follows:

$$\begin{cases} \vec{Q}_2 = \left(l_2 \cos\theta_2\right)\vec{i} + \left(l_2 \sin\theta_2\right)\vec{j}, \\ \vec{Q}_3 = \left(L_2 \cos\theta_2 + l_3 \cos\theta_3\right)\vec{i} + \left(L_2 \sin\theta_2 + l_3 \sin\theta_3\right)\vec{j}, \\ \vec{Q}_4 = \left(L_2 \cos\theta_2 + L_3 \cos\theta_3 + l_4 \cos\theta_4\right)\vec{i} + \left(L_2 \sin\theta_2 + L_3 \sin\theta_3 + l_4 \sin\theta_4\right)\vec{j}. \end{cases} \tag{A3}$$

(b). Taking derivatives of (A3), the velocities of the CoMs are:

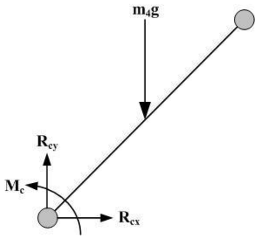
$$\begin{cases} \dot{\vec{Q}}_2 = \left(-l_2\dot{\theta}_2 \sin\theta_2\right)\vec{i} + \left(l_2\dot{\theta}_2 \cos\theta_2\right)\vec{j}, \\ \dot{\vec{Q}}_3 = \left(-L_2\dot{\theta}_2 \sin\theta_2 - l_3\dot{\theta}_3 \sin\theta_3\right)\vec{i} + \left(L_2\dot{\theta}_2 \cos\theta_2 + l_3\dot{\theta}_3 \cos\theta_3\right)\vec{j}, \\ \dot{\vec{Q}}_4 = \left(-L_2\dot{\theta}_2 \sin\theta_2 - L_3\dot{\theta}_3 \sin\theta_3 - l_4\dot{\theta}_4 \sin\theta_4\right)\vec{i} + \left(L_2\dot{\theta}_2 \cos\theta_2 + L_3\dot{\theta}_3 \cos\theta_3 + l_4\dot{\theta}_4 \cos\theta_4\right)\vec{j}. \end{cases} \tag{A4}$$

(c). Taking derivatives of (A4), the accelerations of the CoMs become:

$$\begin{cases} \ddot{\vec{Q}}_2 = \left(-l_2\dot{\theta}_2^2 \cos\theta_2 - l_2\ddot{\theta}_2 \sin\theta_2\right)\vec{i} + \left(-l_2\dot{\theta}_2^2 \sin\theta_2 + l_2\ddot{\theta}_2 \cos\theta_2\right)\vec{j}, \\ \ddot{\vec{Q}}_3 = \left(-L_2\dot{\theta}_2^2 \cos\theta_2 - L_2\ddot{\theta}_2 \sin\theta_2 - l_3\dot{\theta}_3^2 \cos\theta_3 - l_3\ddot{\theta}_3 \sin\theta_3\right)\vec{i} \\ \qquad + \left(-L_2\dot{\theta}_2^2 \sin\theta_2 + L_2\ddot{\theta}_2 \cos\theta_2 - l_3\dot{\theta}_3^2 \sin\theta_3 + l_3\ddot{\theta}_3 \cos\theta_3\right)\vec{j}, \\ \ddot{\vec{Q}}4 = \left(-L_2\dot{\theta}_2^2 \cos\theta_2 - L_2\ddot{\theta}_2 \sin\theta_2 - L_3\dot{\theta}_3^2 \cos\theta_3 - L_3\ddot{\theta}_3 \sin\theta_3 - l_4\dot{\theta}_4^2 \cos\theta_4 - l_4\ddot{\theta}_4 \sin\theta_4\right)\vec{i} \\ \qquad + \left(-L_2\dot{\theta}_2^2 \sin\theta_2 + L_2\ddot{\theta}_2 \cos\theta_2 - L_3\dot{\theta}_3^2 \sin\theta_3 + L_3\ddot{\theta}_3 \cos\theta_3 - l_4\dot{\theta}_4^2 \sin\theta_4 + l_4\ddot{\theta}_4 \cos\theta_4\right)\vec{j}. \end{cases}$$

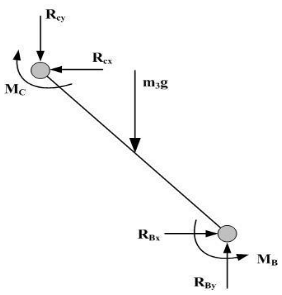
Applying Newton's Second Law ($\Sigma F = Ma$, $\Sigma M = I\alpha$) to the free-body diagrams of the segments results in the following dynamic equations:

(1). Trunk



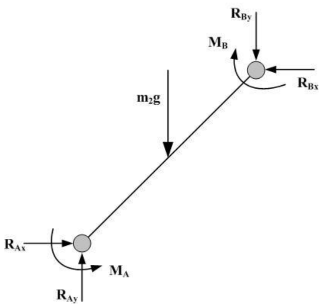
$$
\begin{aligned}
&\Sigma F_X : R_{Cx} = m_4 \cdot \ddot{Q}_{4x}, \\
&\Sigma F_Y : R_{Cy} - m_4 g = m_4 \cdot \ddot{Q}_{4y}, \\
&\Sigma M_{Q_4} : M_c + R_{cx} \cdot l_4 \sin\theta_4 - R_{cy} \cdot l_4 \cos\theta_4 = I_4 \cdot \ddot{\theta}_4.
\end{aligned}
$$

(2). Thigh



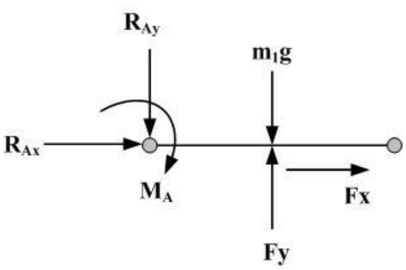
$$
\begin{aligned}
&\Sigma F_X : R_{Bx} - R_{Cx} = m_3 \cdot \ddot{Q}_{3x}, \\
&\Sigma F_Y : R_{By} - R_{Cy} - m_3 g = m_3 \cdot \ddot{Q}_{3y}, \\
&\Sigma M_{Q_3} : M_B - M_C + R_{Cy}(L_3 - l_3)\cos\theta_3 + R_{Cx}(L_3 - l_3)\sin\theta_3 \\
&\qquad + R_{By} l_3 \cos\theta_3 + R_{Bx} l_3 \sin\theta_3 = I_3 \cdot \ddot{\theta}_3.
\end{aligned}
$$

(3). Shank

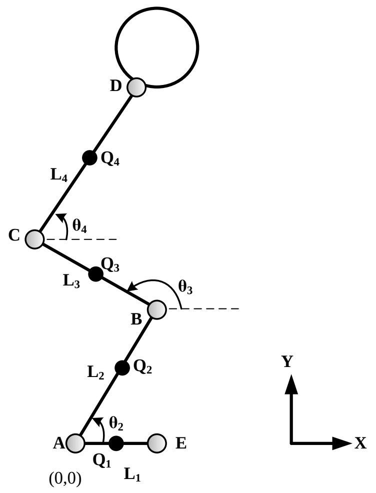


$$
\begin{aligned}
&\Sigma F_X : R_{Ax} - R_{Bx} = m_2 \cdot \ddot{Q}_{2x}, \\
&\Sigma F_Y : R_{Ay} - R_{By} - m_2 g = m_2 \cdot \ddot{Q}_{2y}, \\
&\Sigma M_A : M_A - M_B + R_{Bx} L_2 \sin\theta_2 - m_2 g l_2 \cos\theta_2 \\
&\qquad - R_{By} L_2 \cos\theta_2 = I_A \cdot \ddot{\theta}_2.
\end{aligned}
$$

(4). Foot (Assume the foot was static)



$$
\begin{aligned}
&\Sigma F_X : F_x - R_{Ax} = 0, \\
&\Sigma F_Y : F_y - R_{Ay} - m_1 g = 0, \\
&\Sigma M_A : (F_y - m_1 g) l_1 - M_A = 0.
\end{aligned}
$$



	Line	CoM	Length	Angle
Foot	$\overline{AE}$	Q_1	L_1	
Leg	$\overline{AB}$	Q_2	L_2	θ_2
Thigh	$\overline{BC}$	Q_3	L_3	θ_3
Trunk	$\overline{CD}$	Q_4	L_4	θ_4
$\overline{AQ_1} = l_1, \ \overline{AQ_2} = l_2, \ \overline{BQ_3} = l_3, \ \overline{CQ_4} = l_4.$				

Fig. A1. Derivation of the 2D STS dynamic model.